Learned Wyner–Ziv Compressors Recover Binning

Ezgi Ozyilkan

2023 IEEE International Symposium on Information Theory (ISIT)
Taipei, Taiwan | June 25-30 2023

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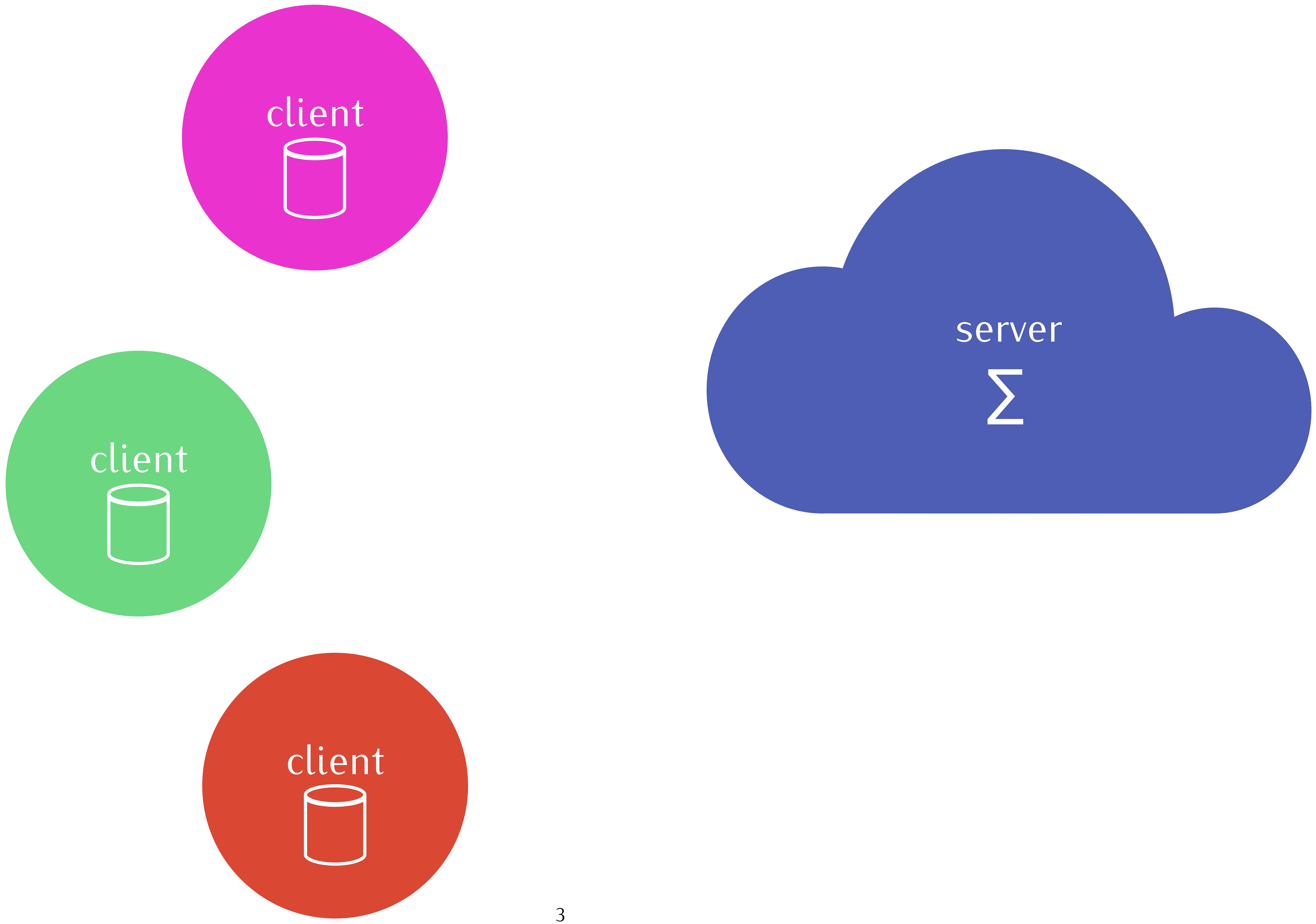
NYU | TANDON SCHOOL OF ENGINEERING

Distributed Source Coding

Motivation: Distributed Source Coding

e.g., next-word prediction



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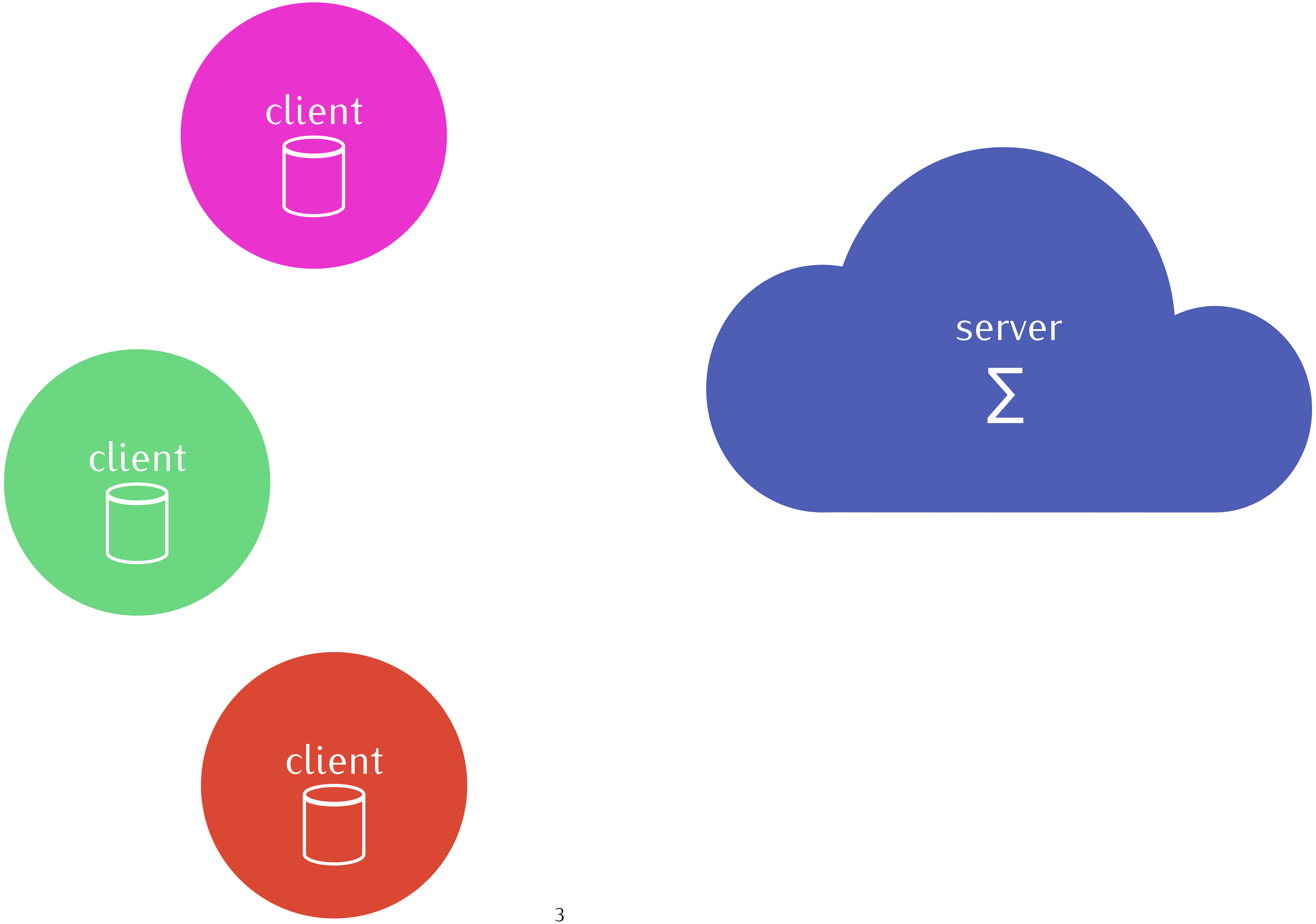

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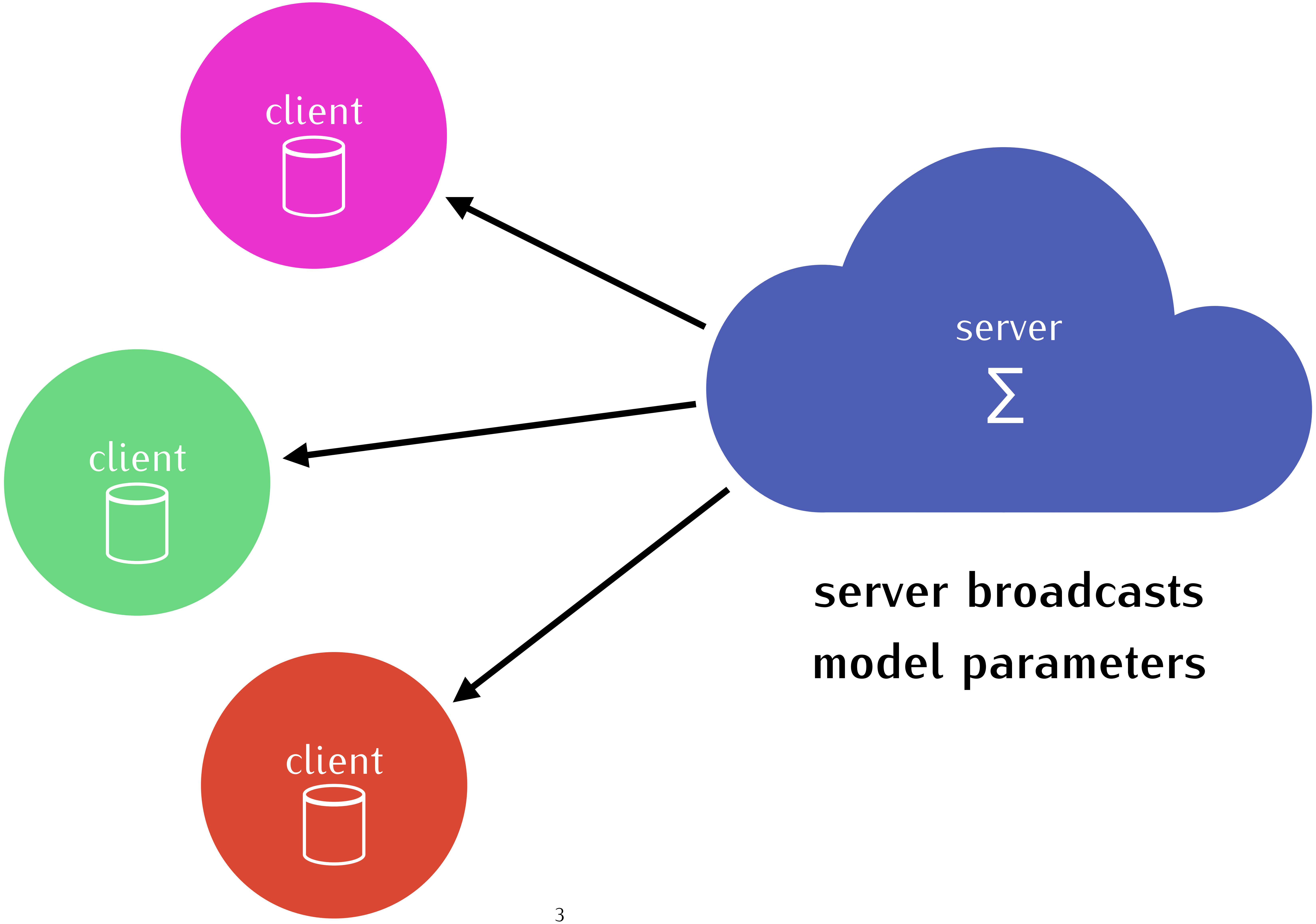

server broadcasts model parameters

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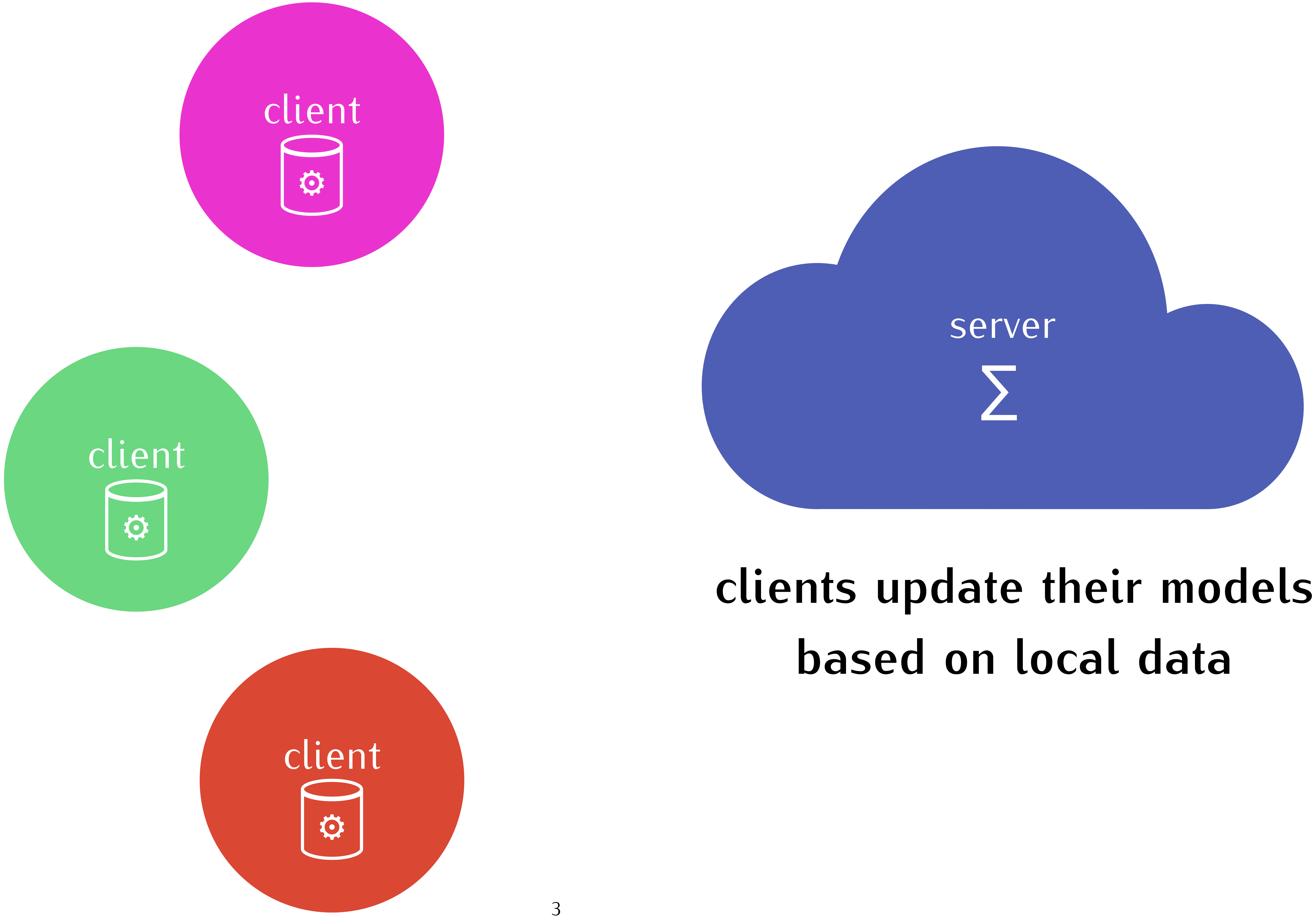

clients update their models based on local data

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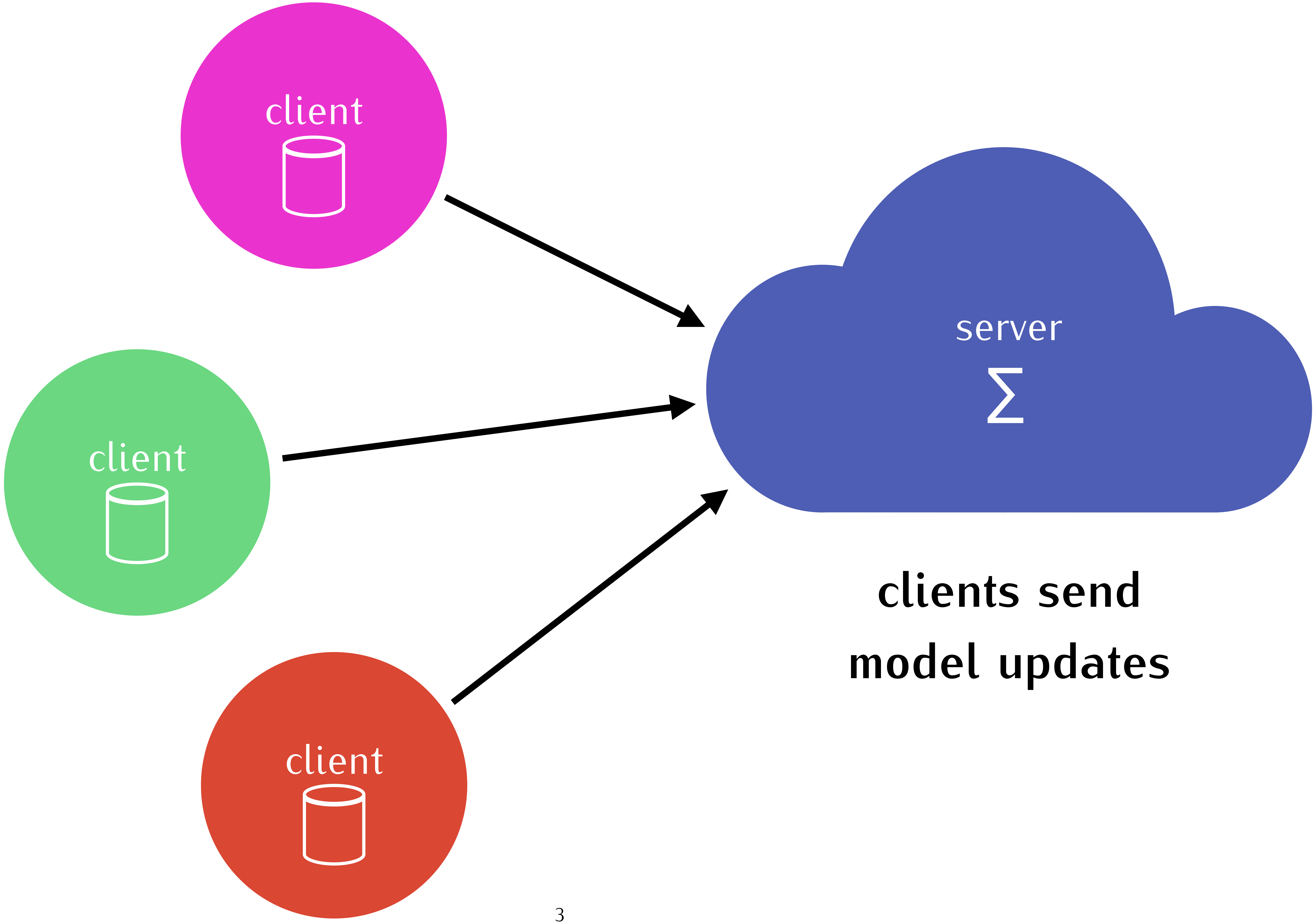

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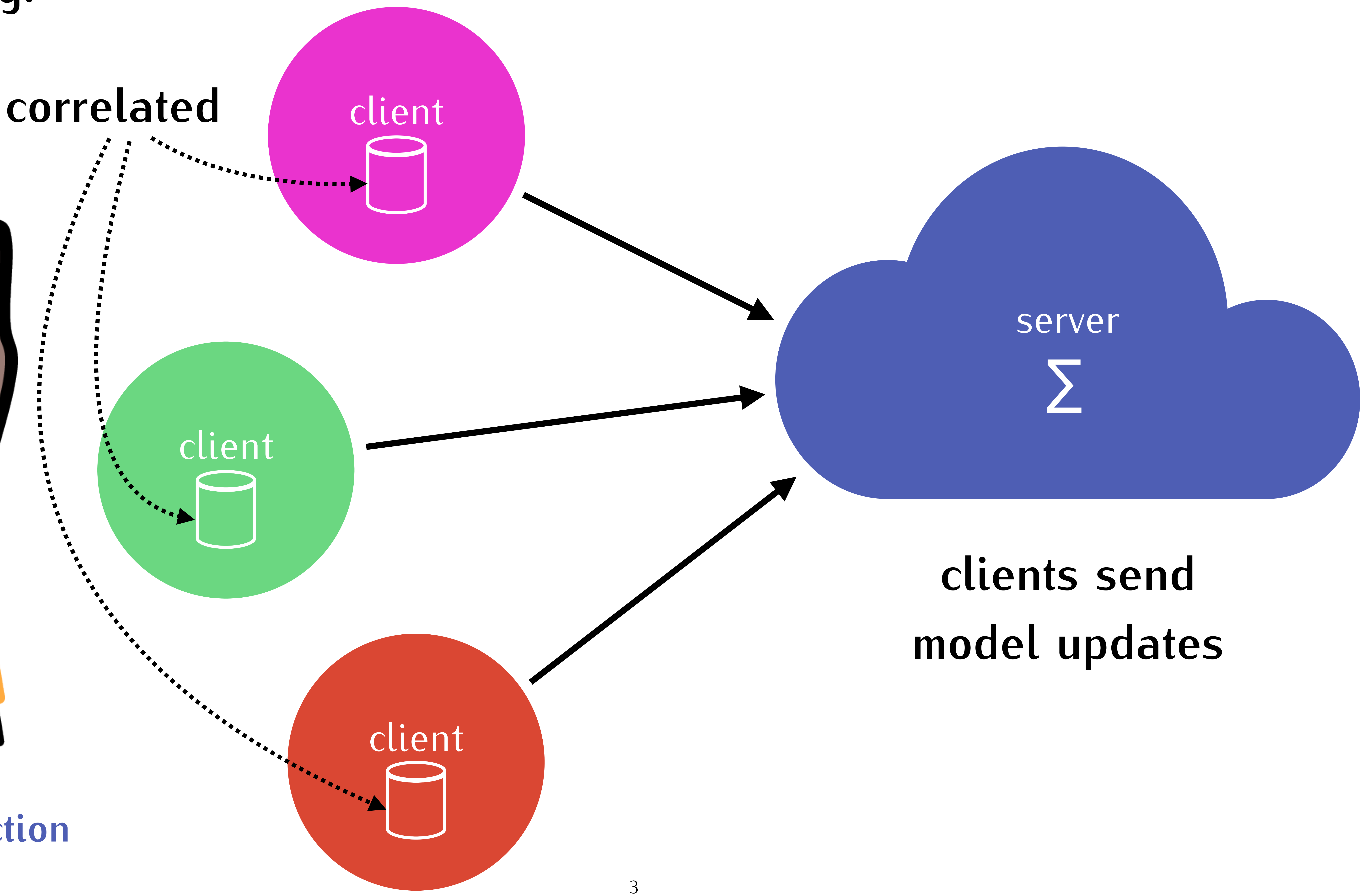

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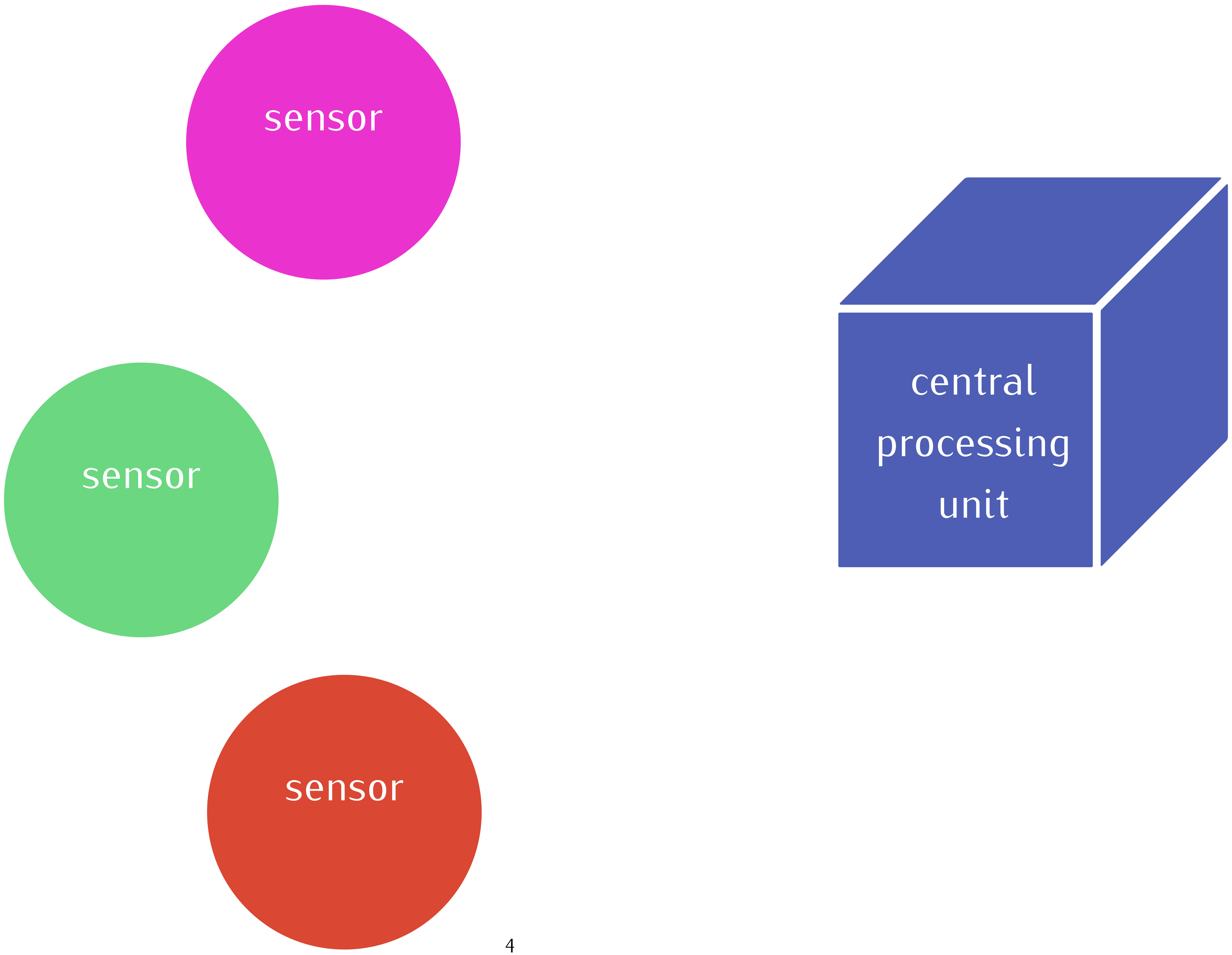
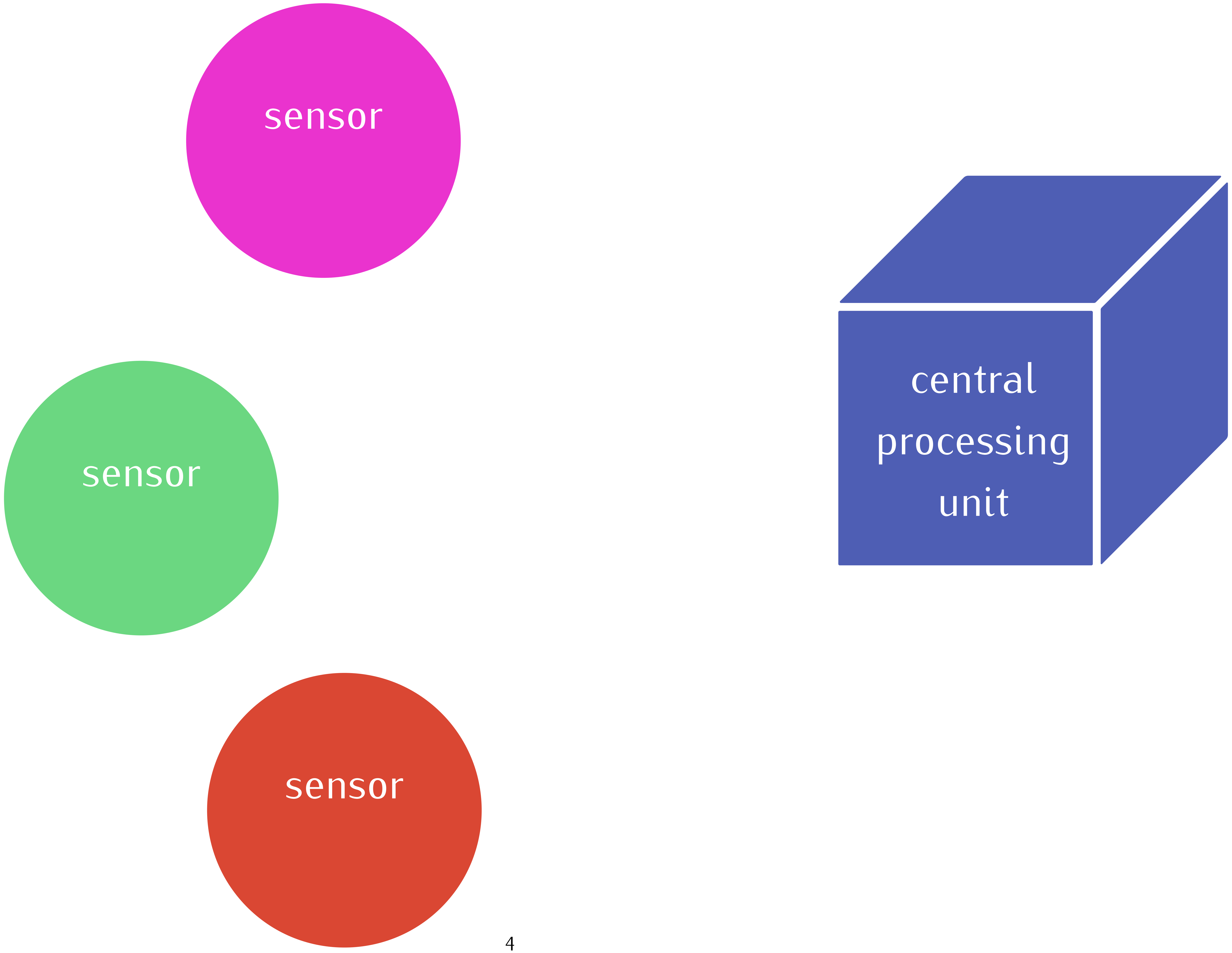


Motivation: Distributed Source Coding

Sensor networks.



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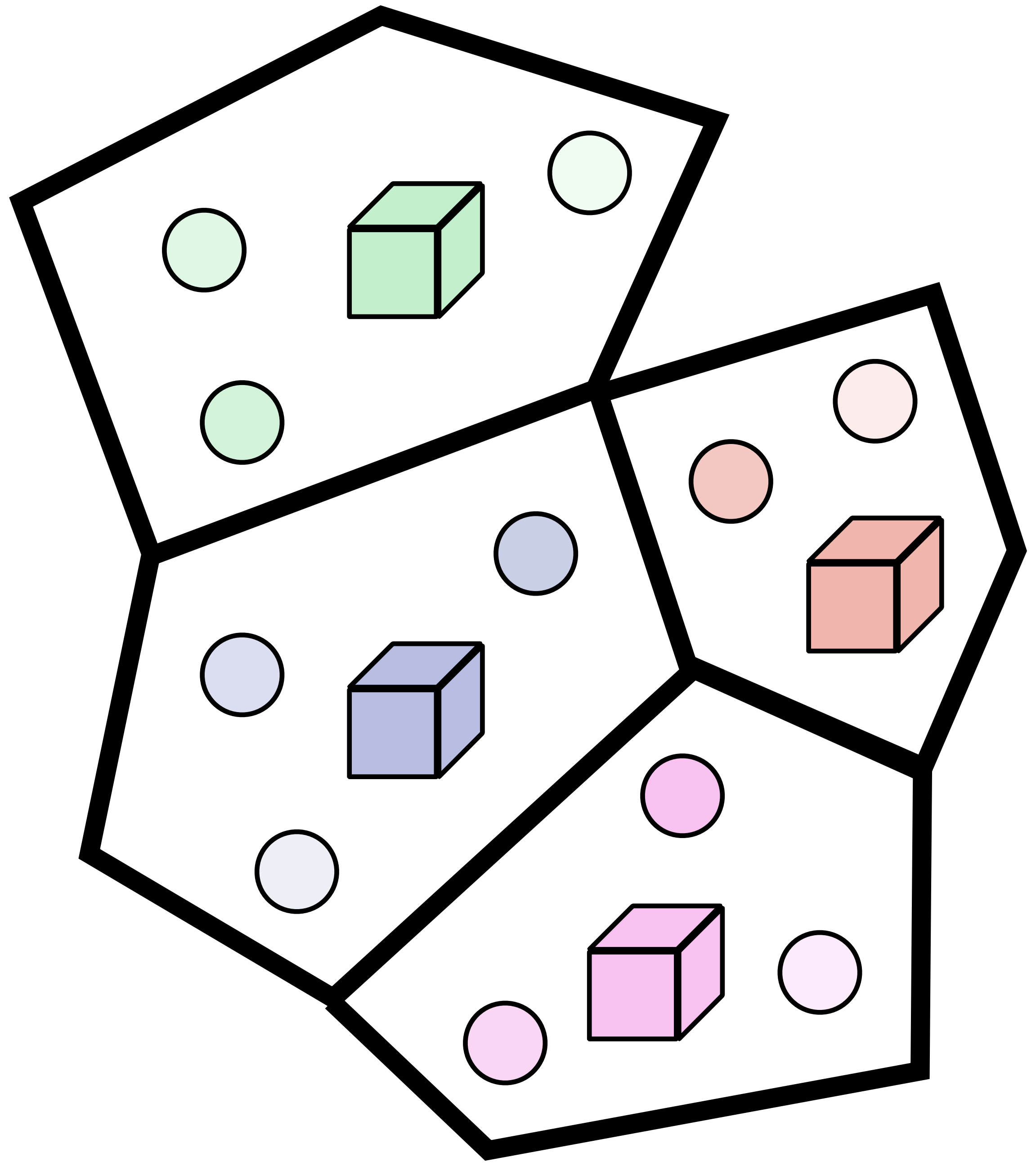

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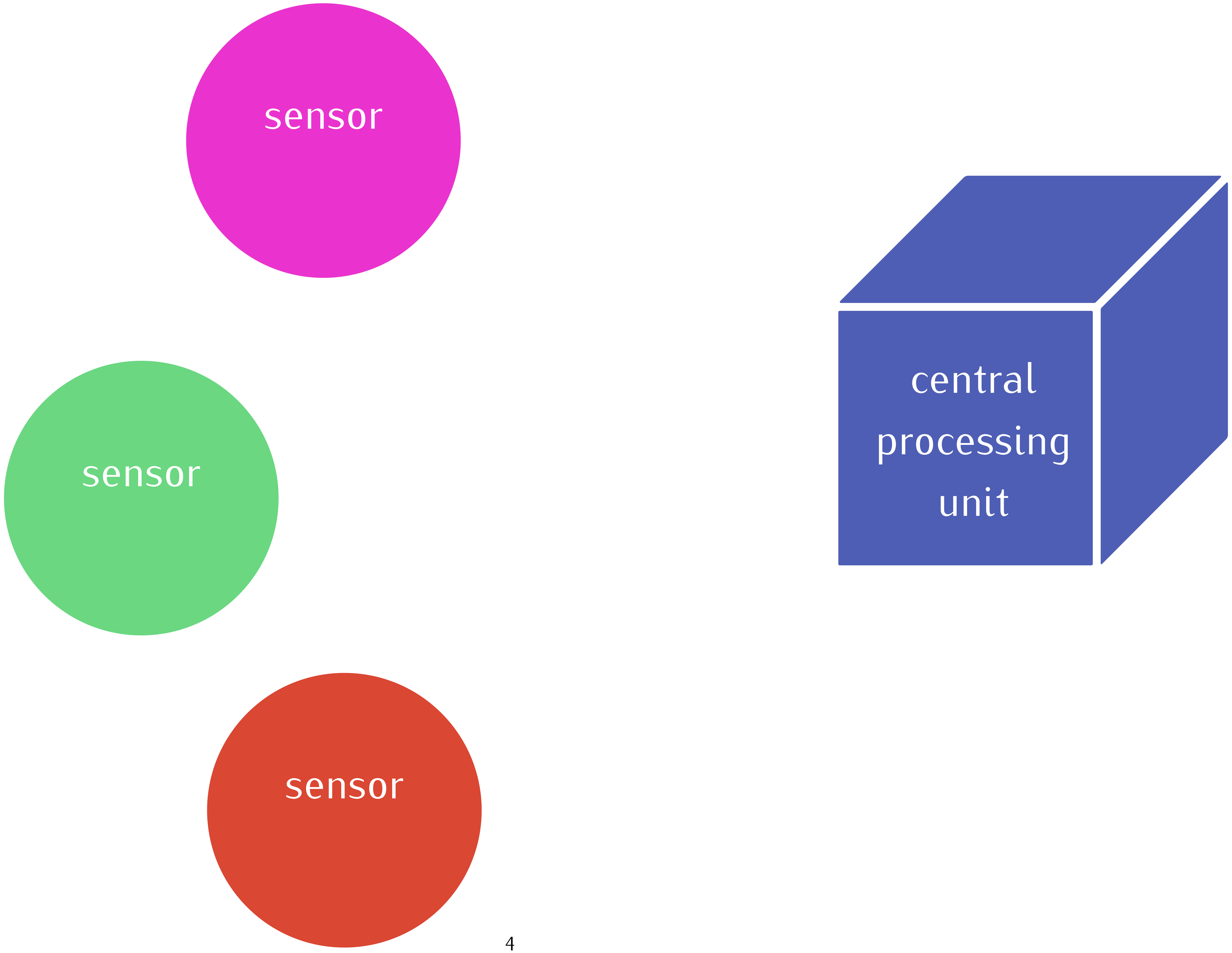

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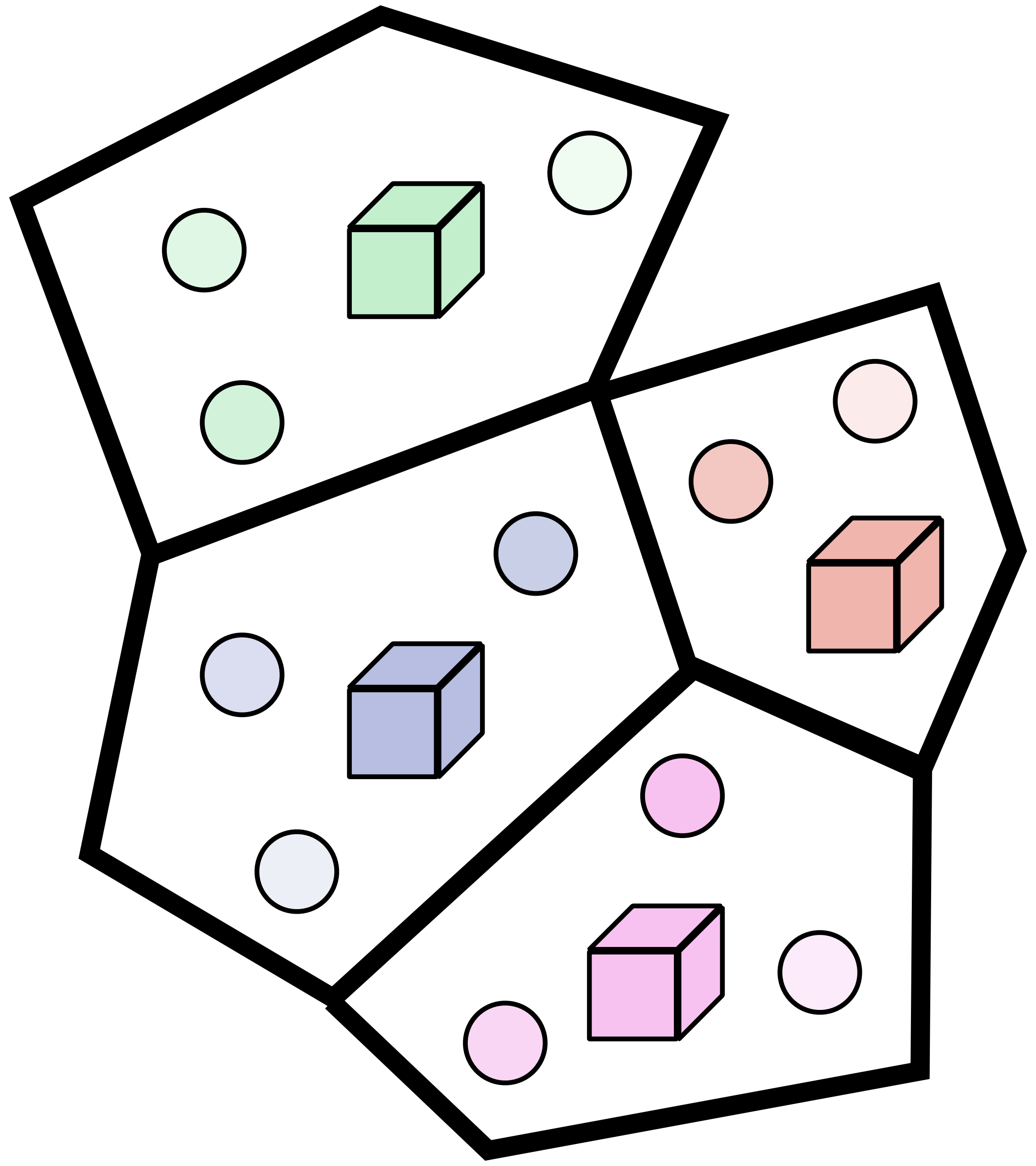

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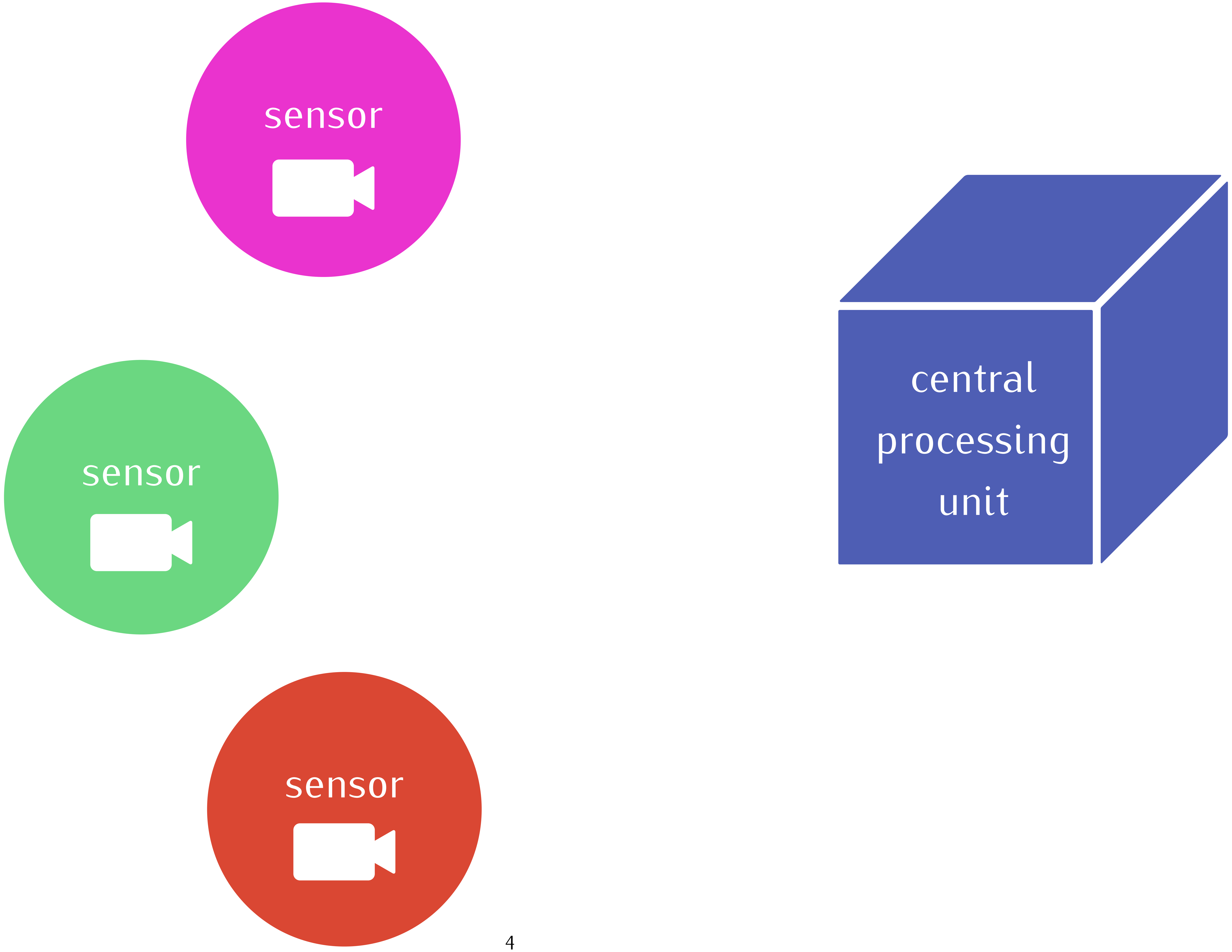

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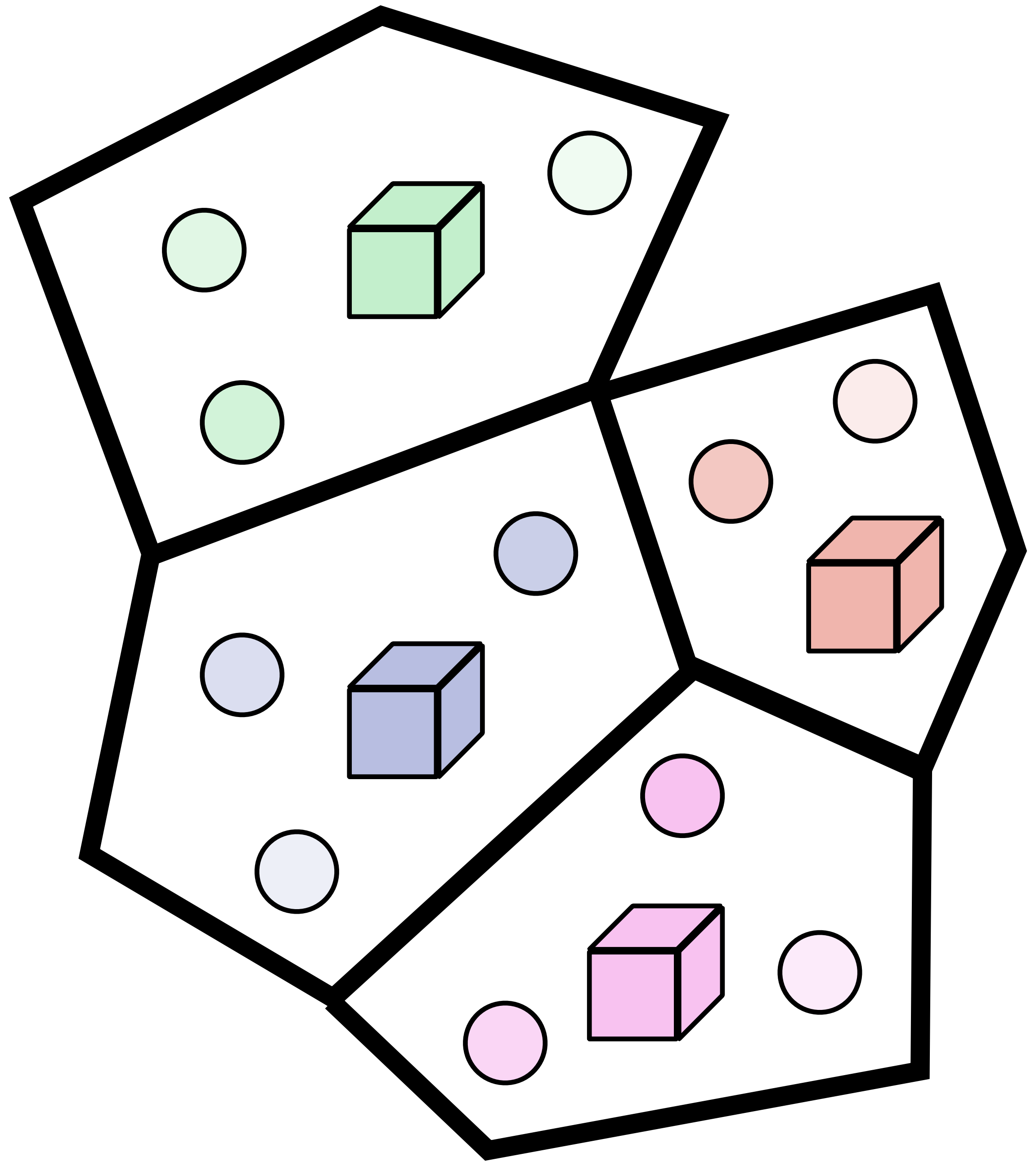

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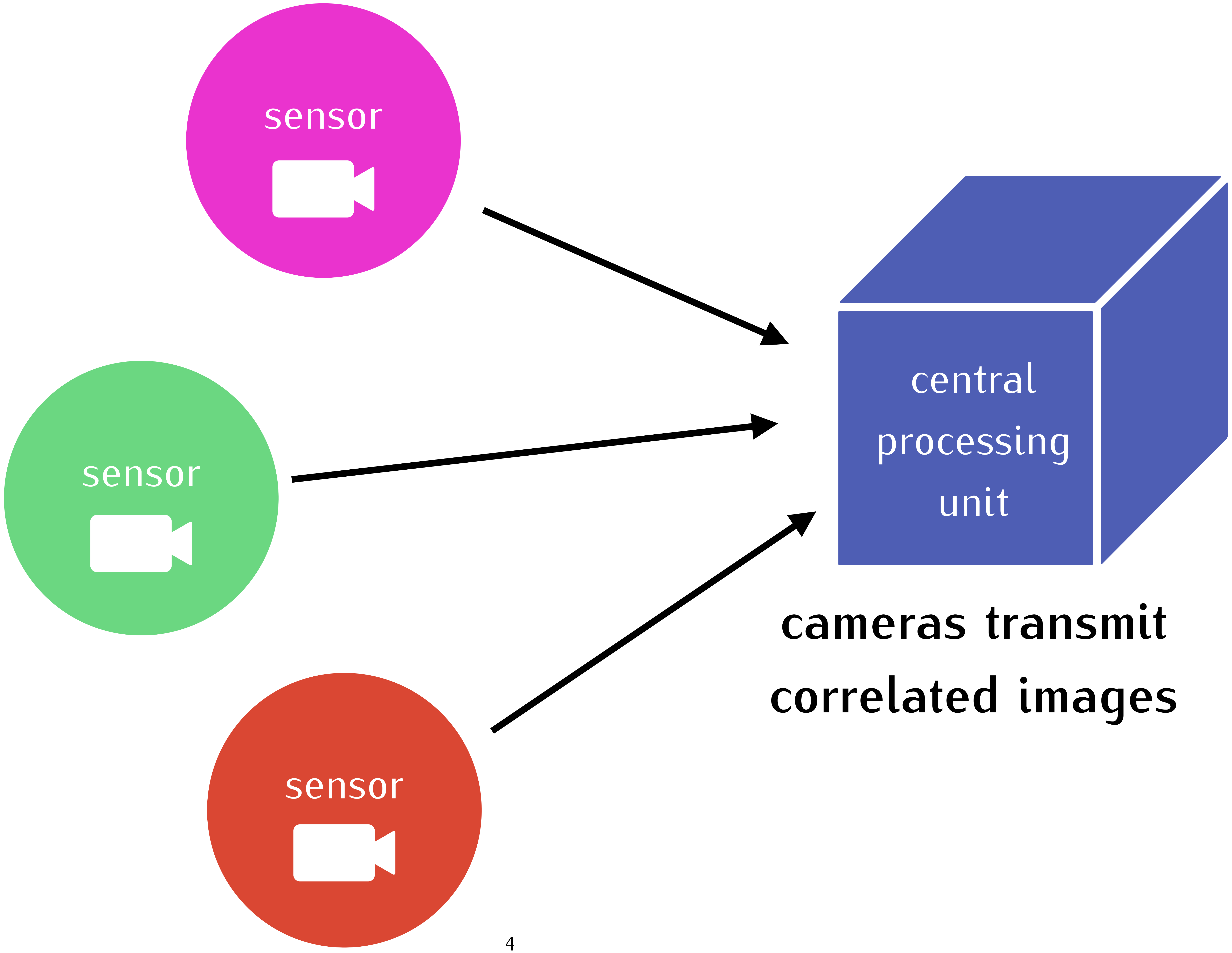

cameras transmit correlated images

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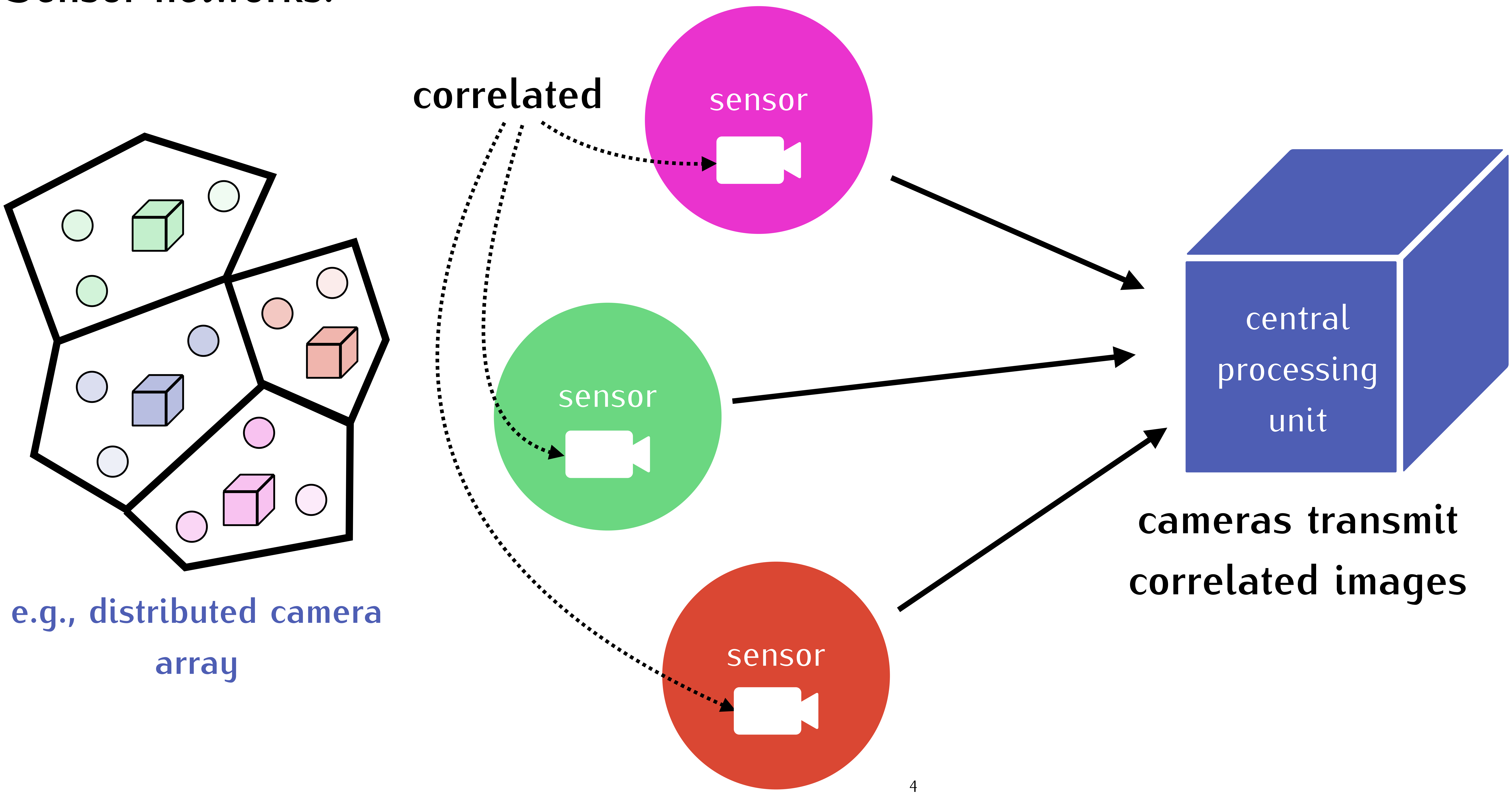

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Learning-based compressors (e.g., Ballé et al., 2017) may help.

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→ Johannes Ballé's keynote at DCC'23.

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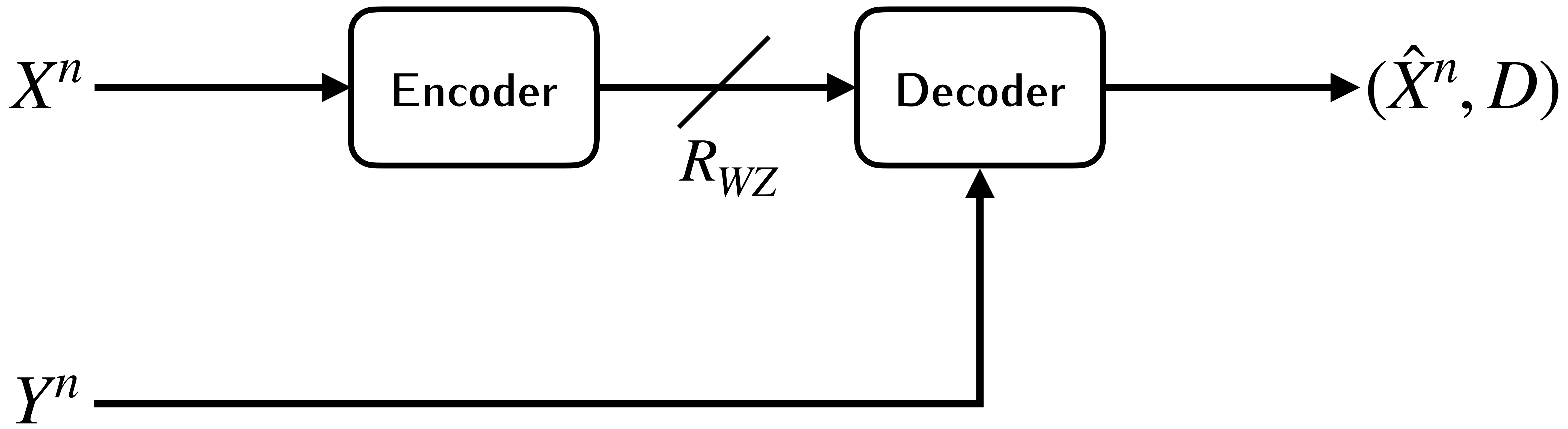

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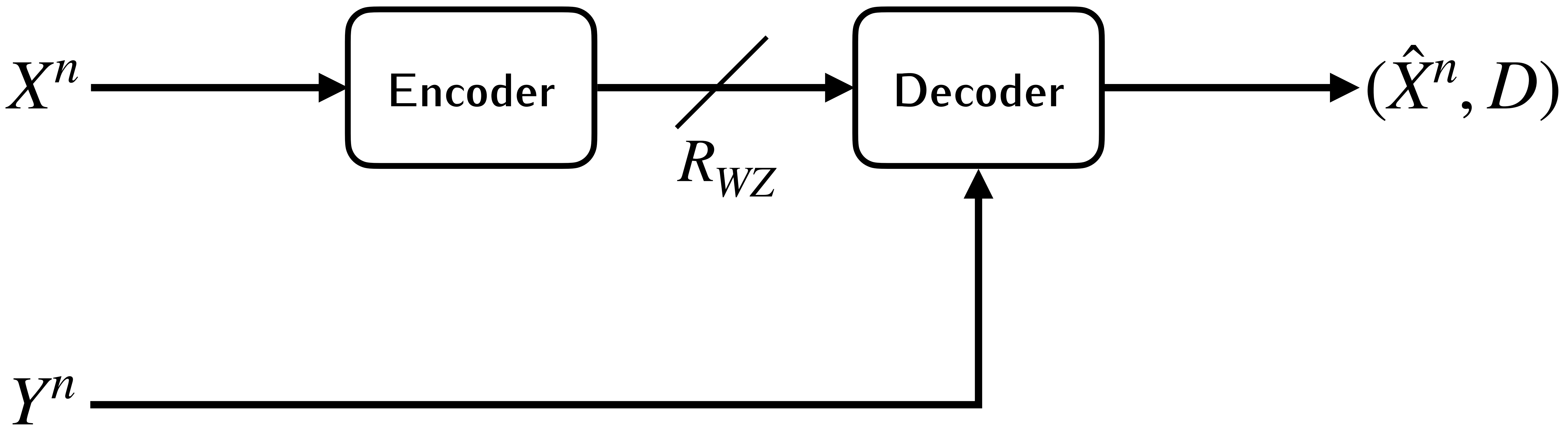

<u>**Theorem.**</u> Let (X, Y) be correlated i.i.d. $\sim p(x, y)$, and let $d(x, \hat{x})$ be a distortion measure. The R-D function for X when Y available at the decoder is:

$$R_{WZ}(D) = \min(I(X;U) - I(Y;U)),$$

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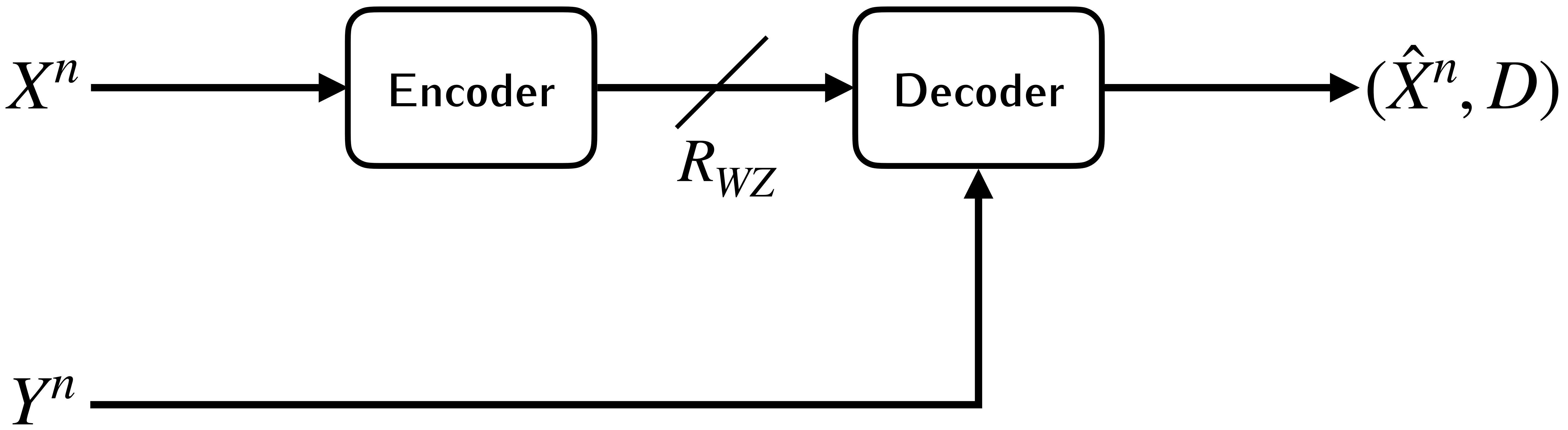

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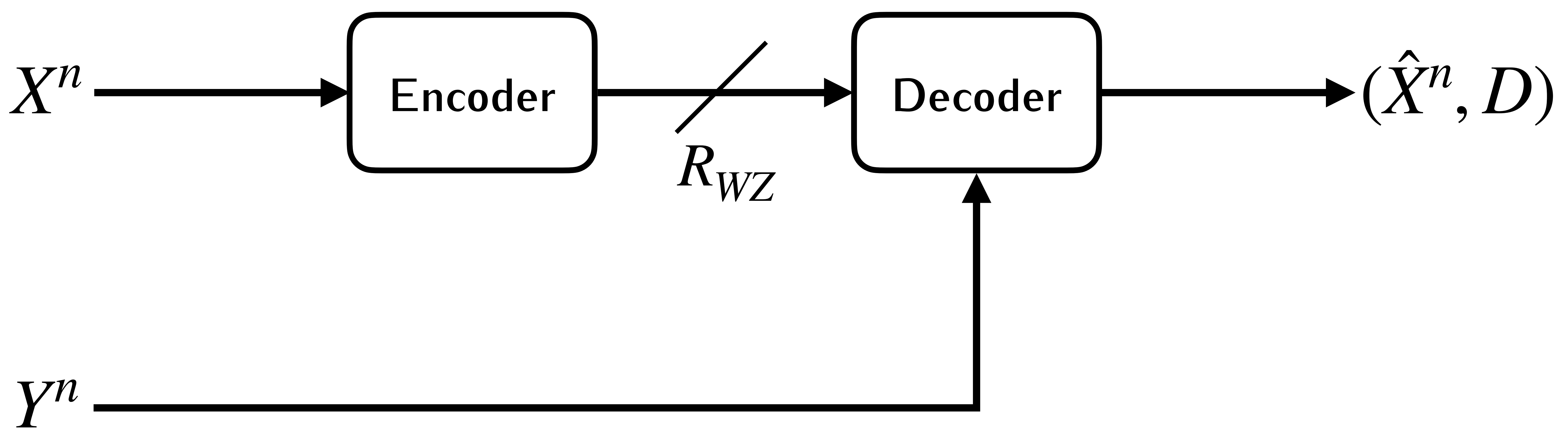

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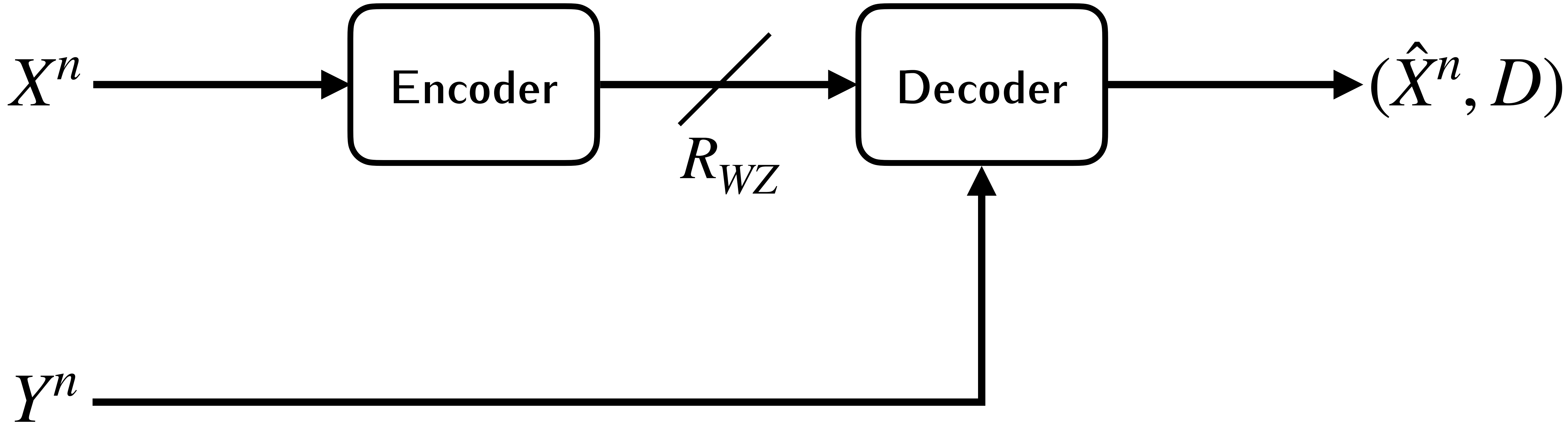

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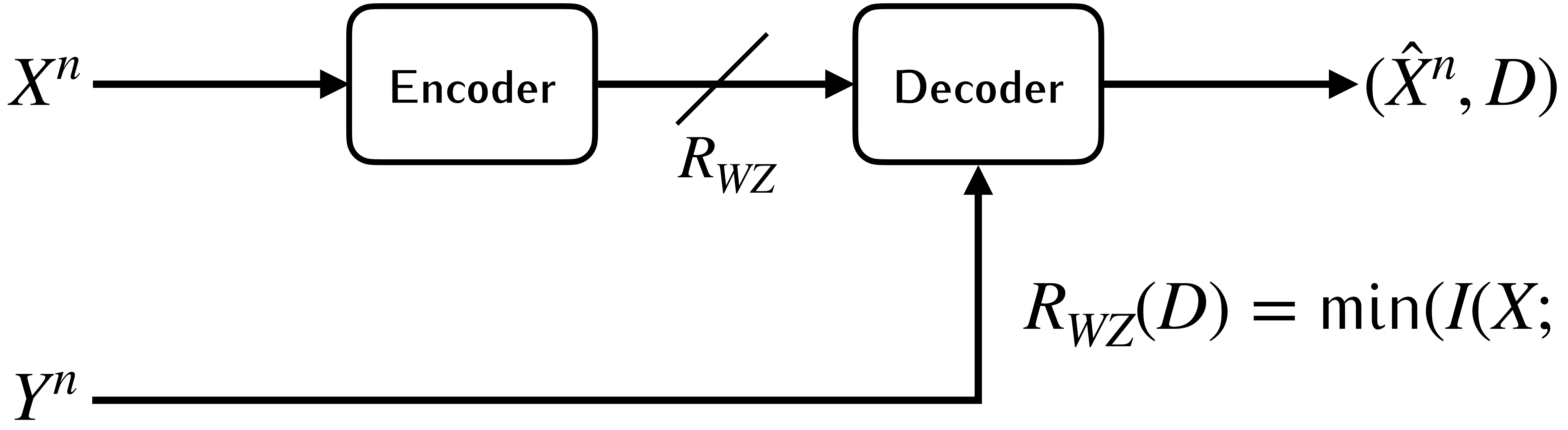

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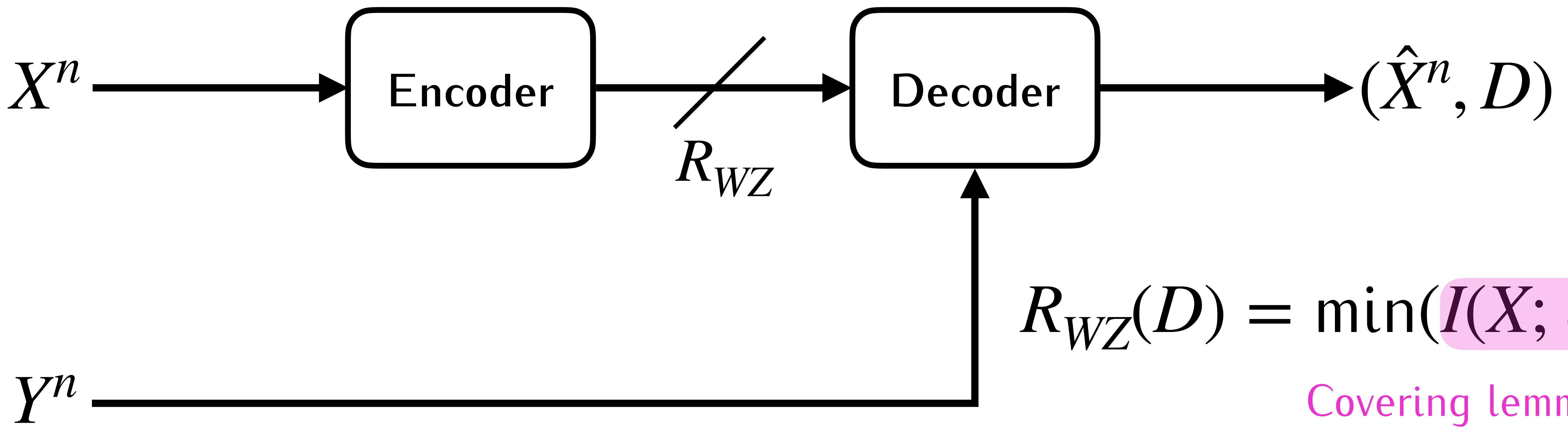

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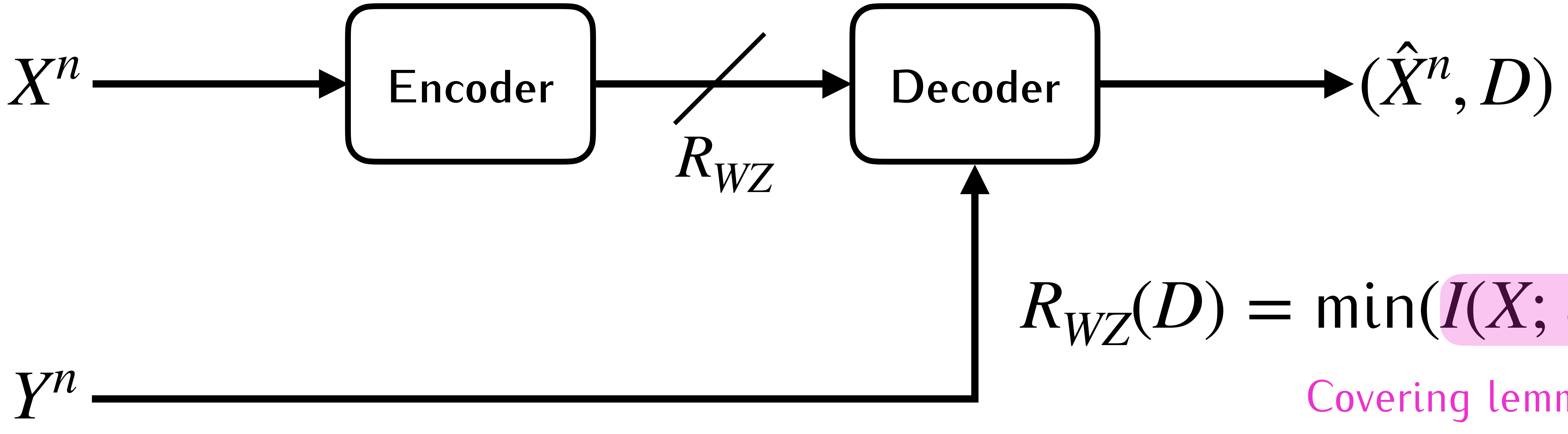

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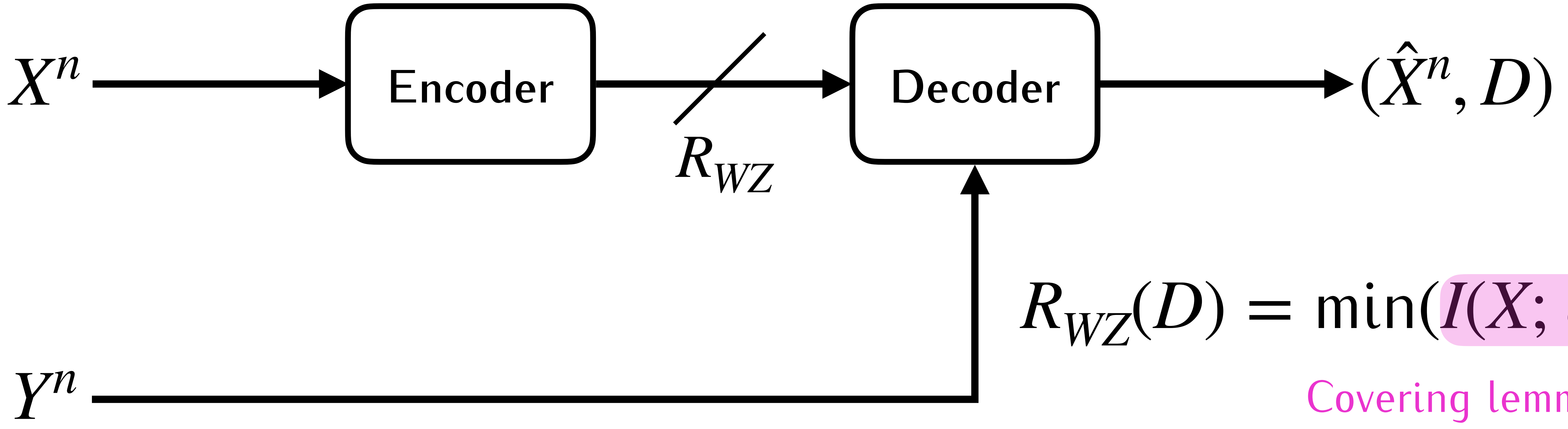

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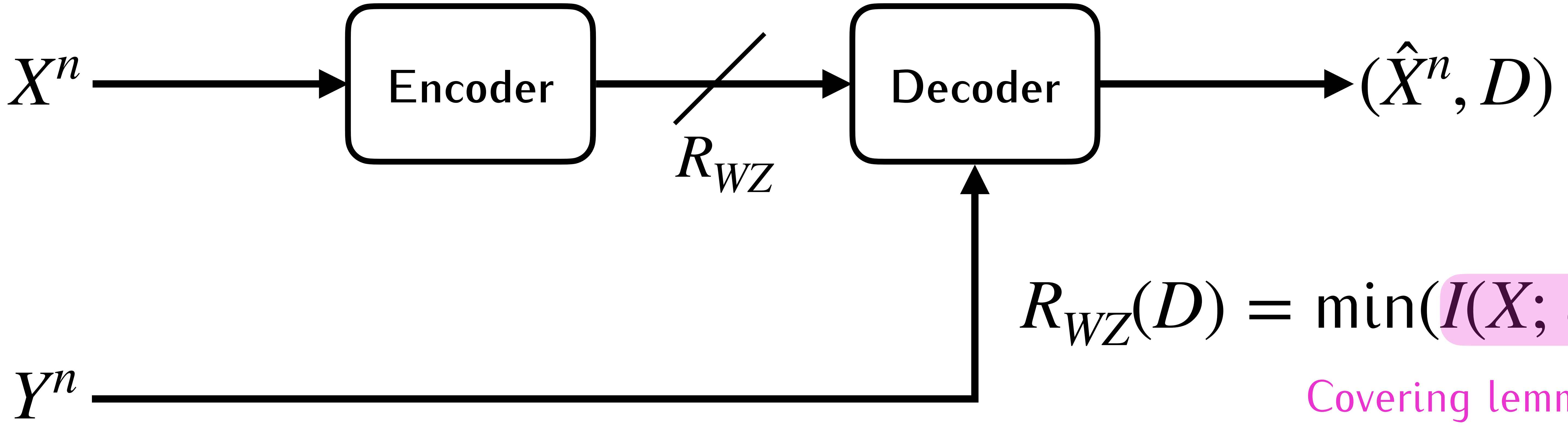

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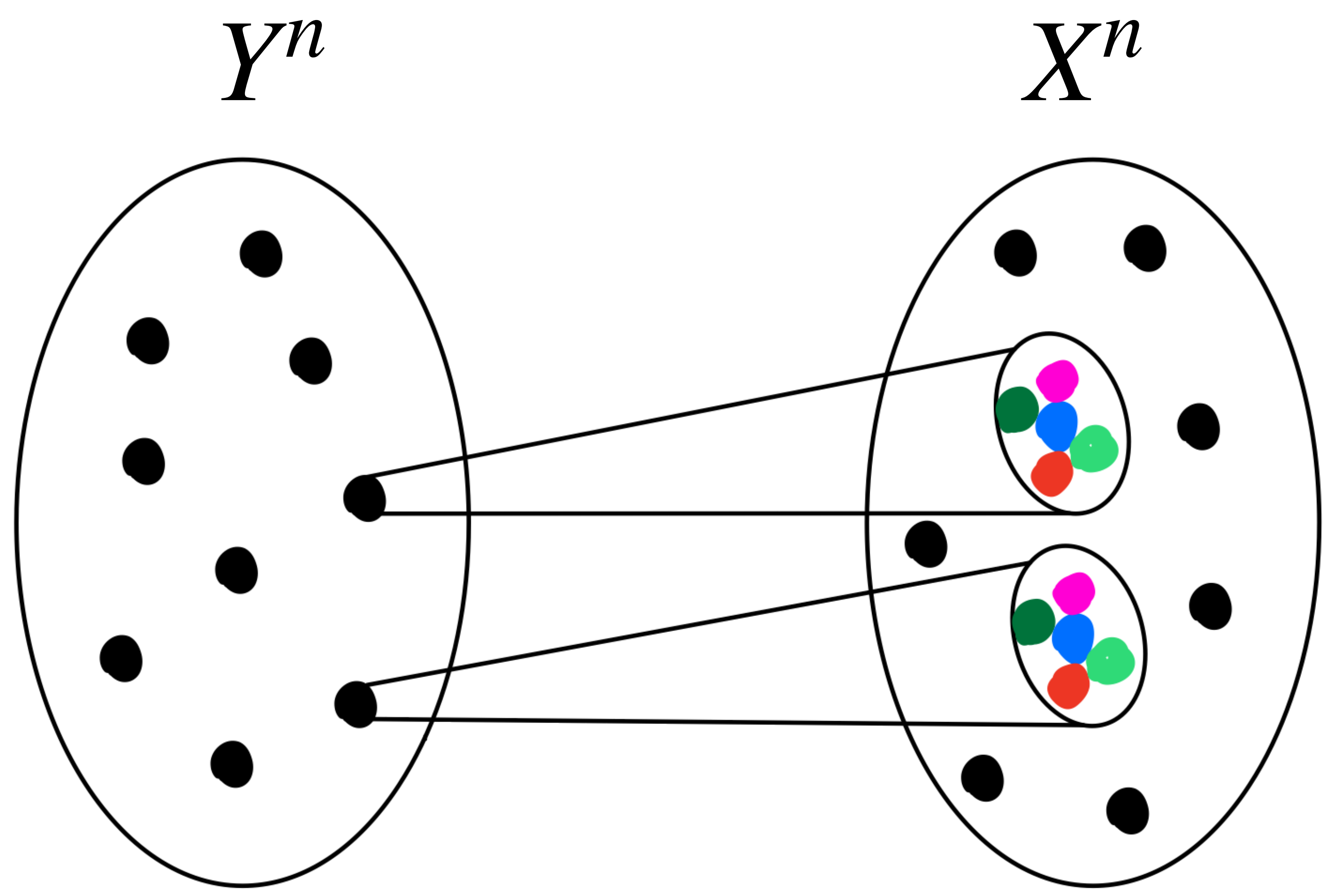

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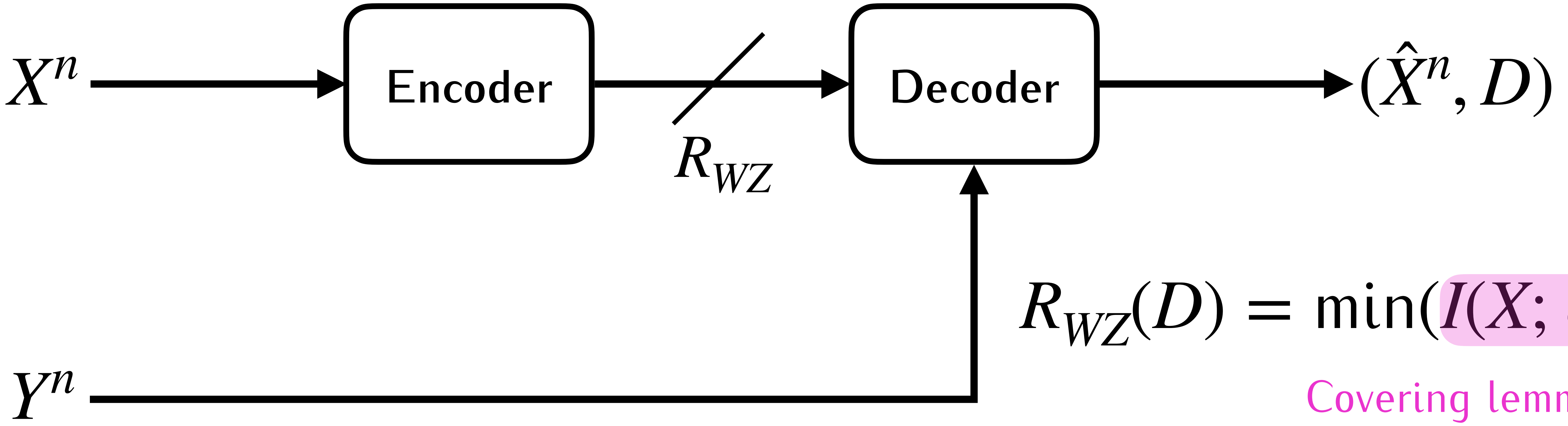

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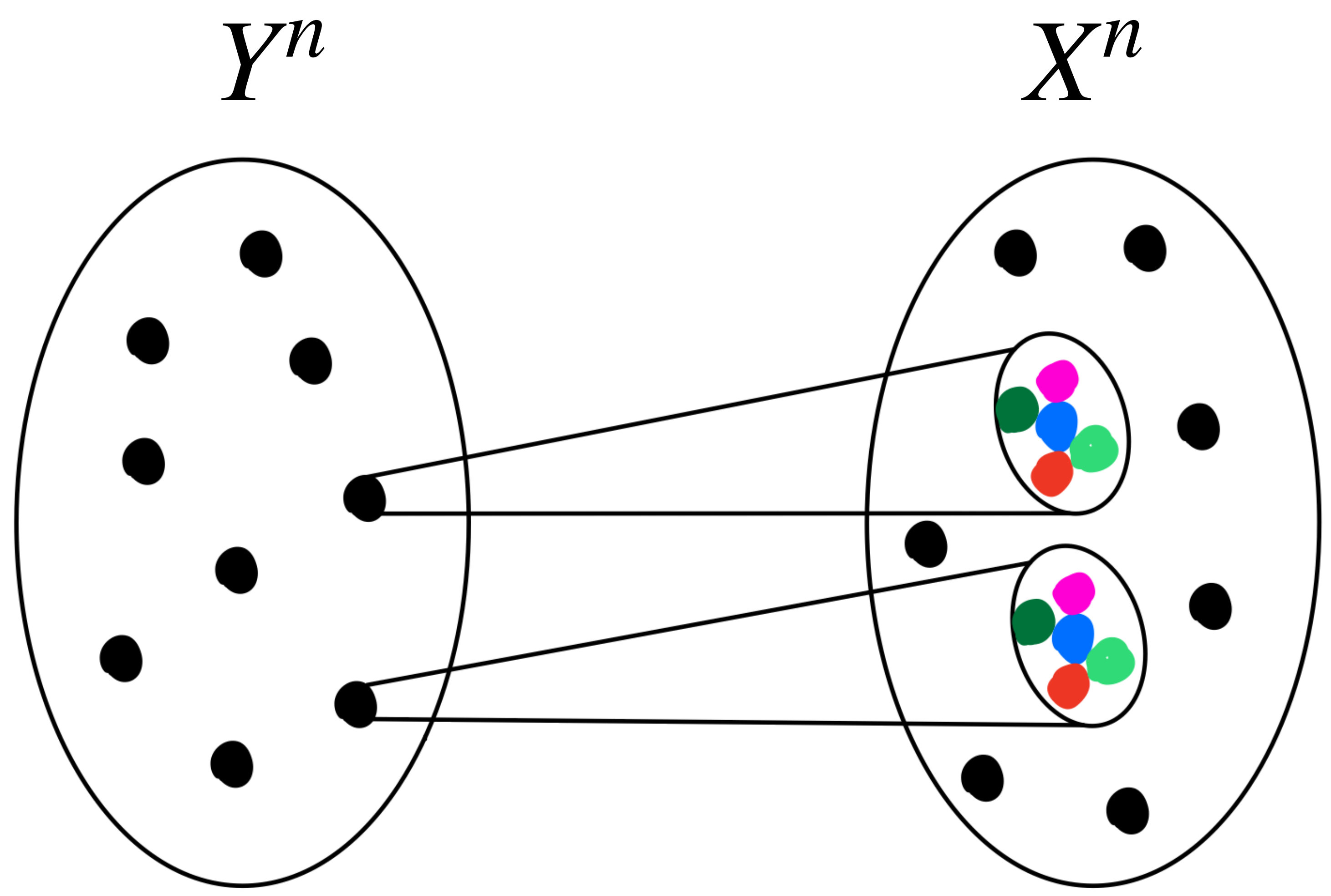

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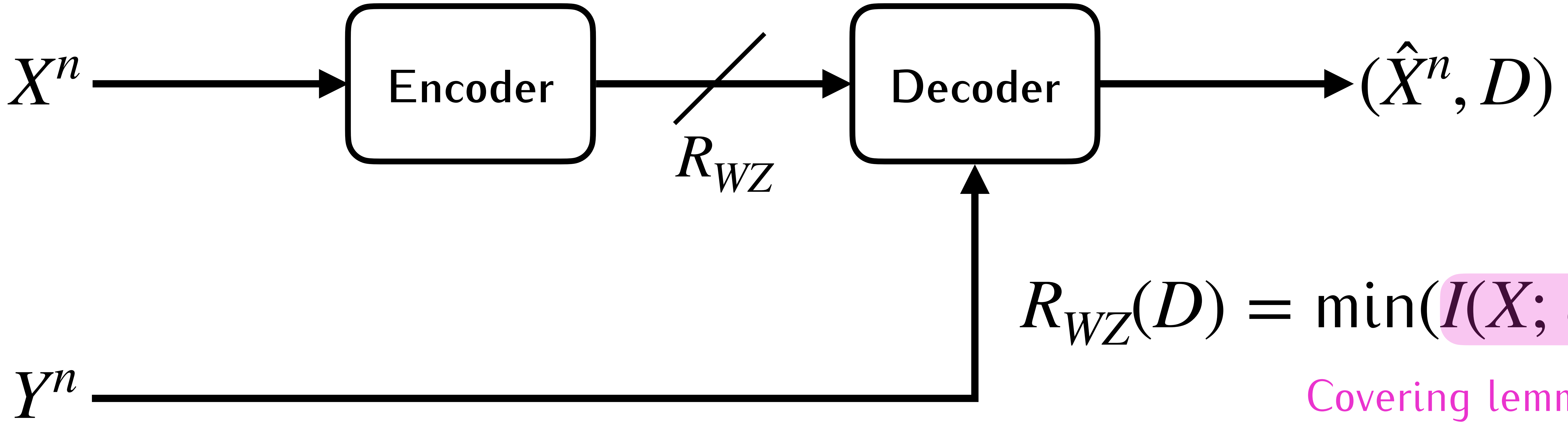

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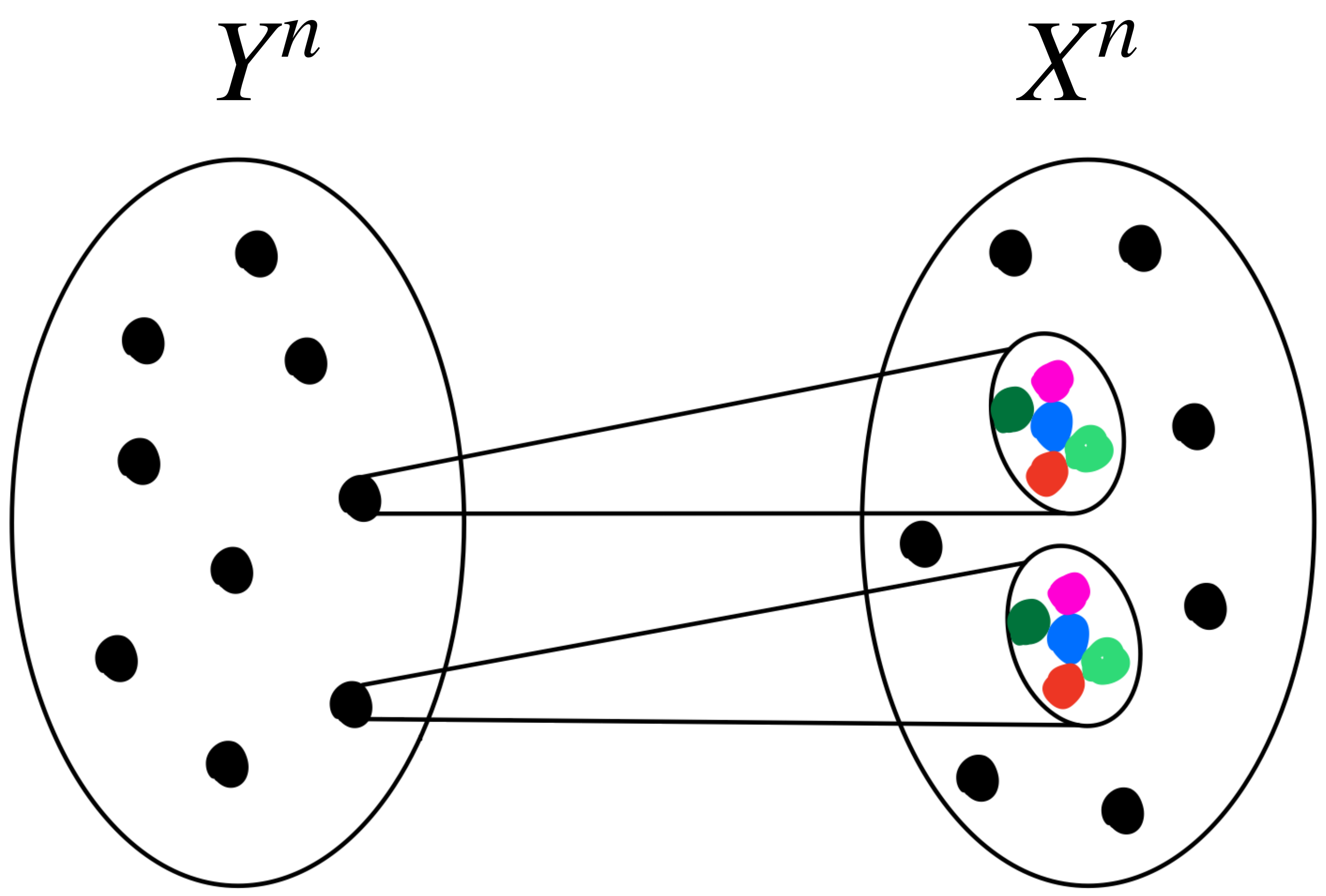

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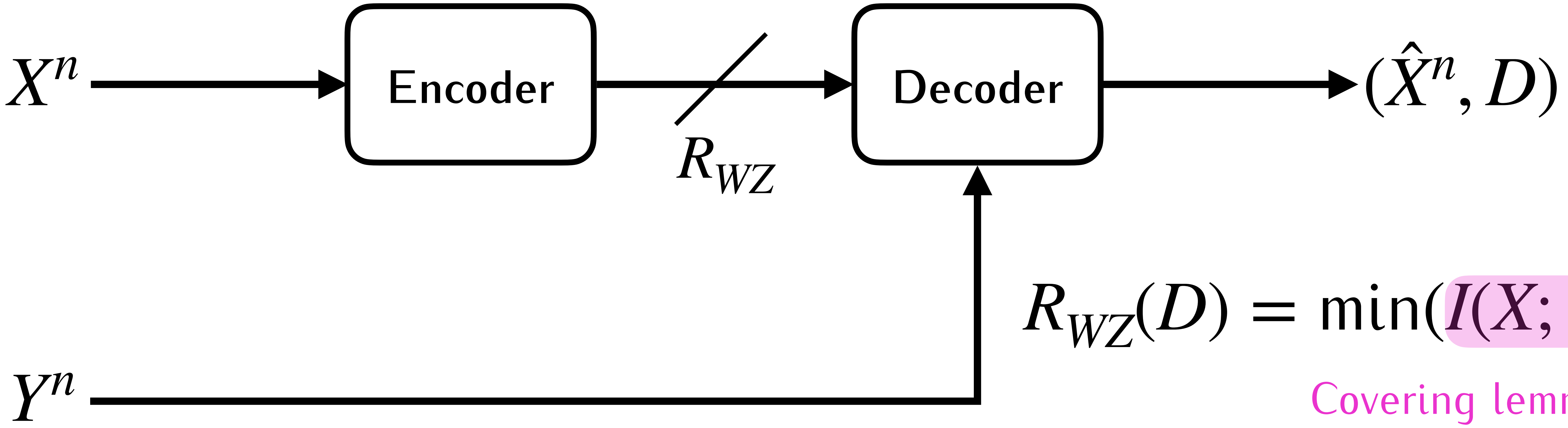

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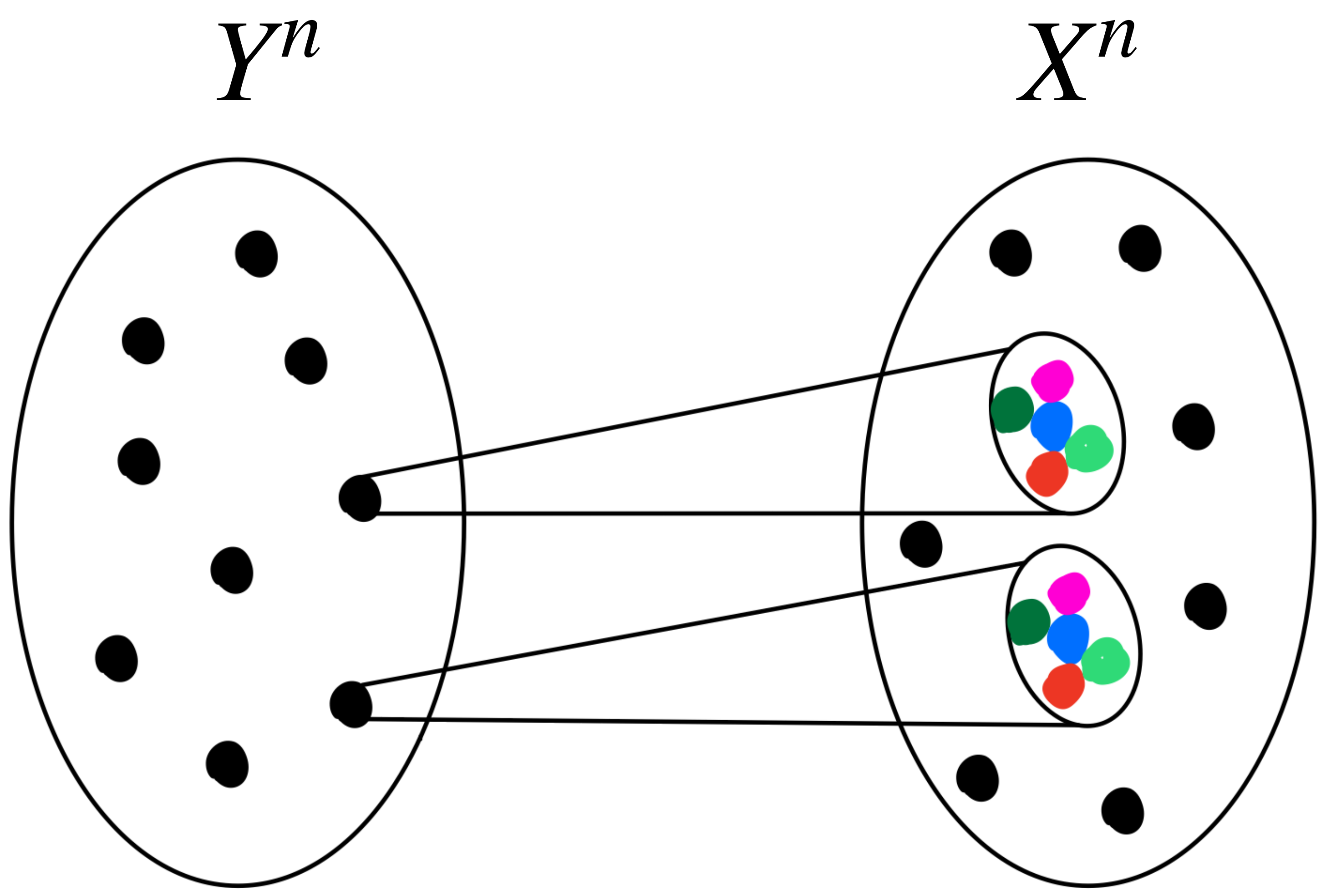

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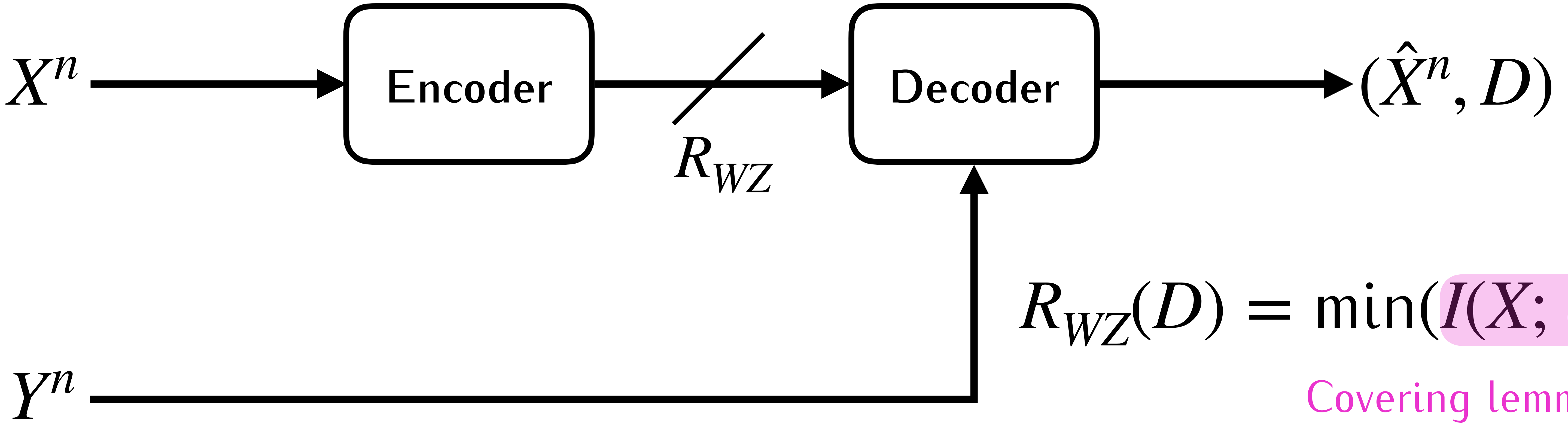

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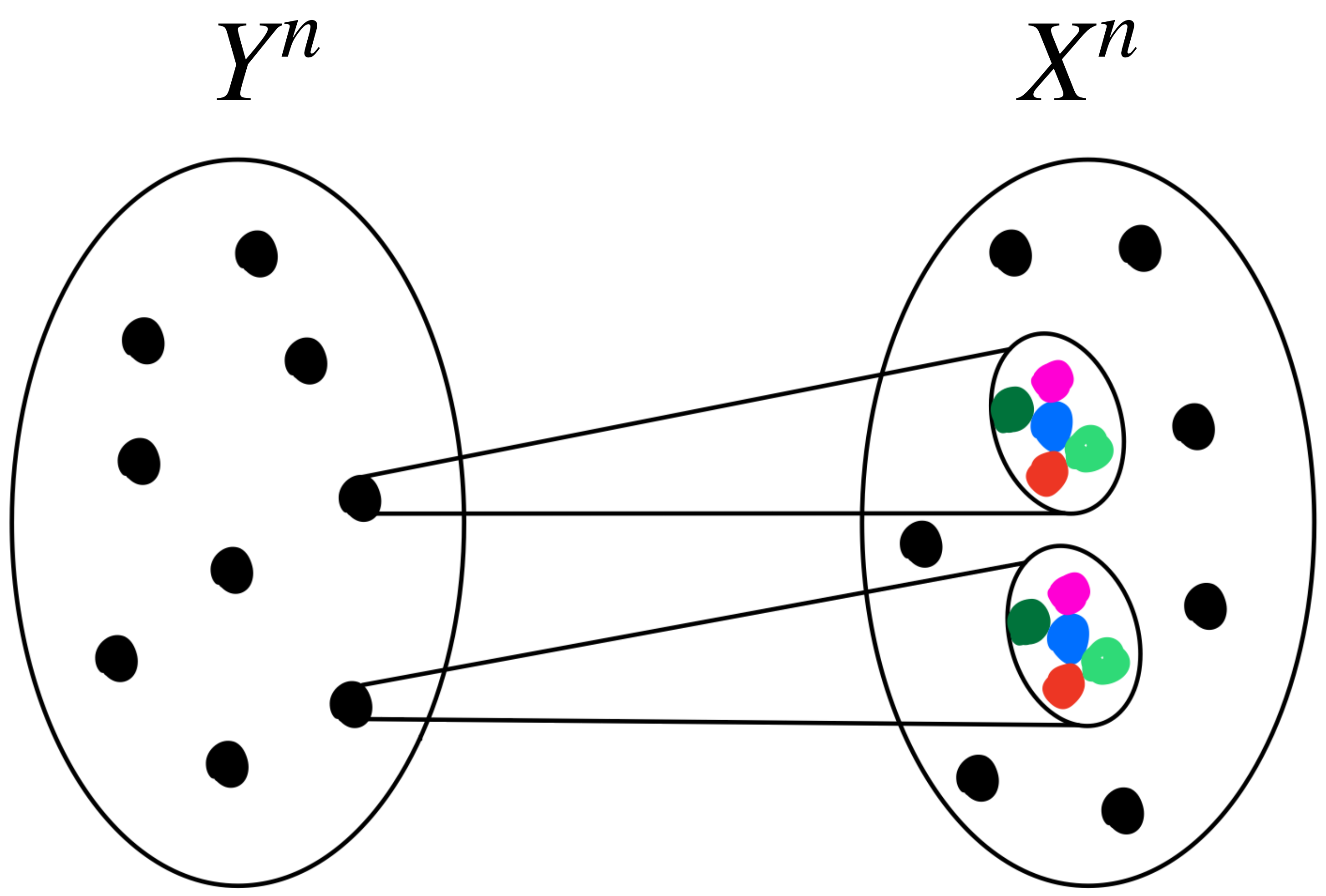

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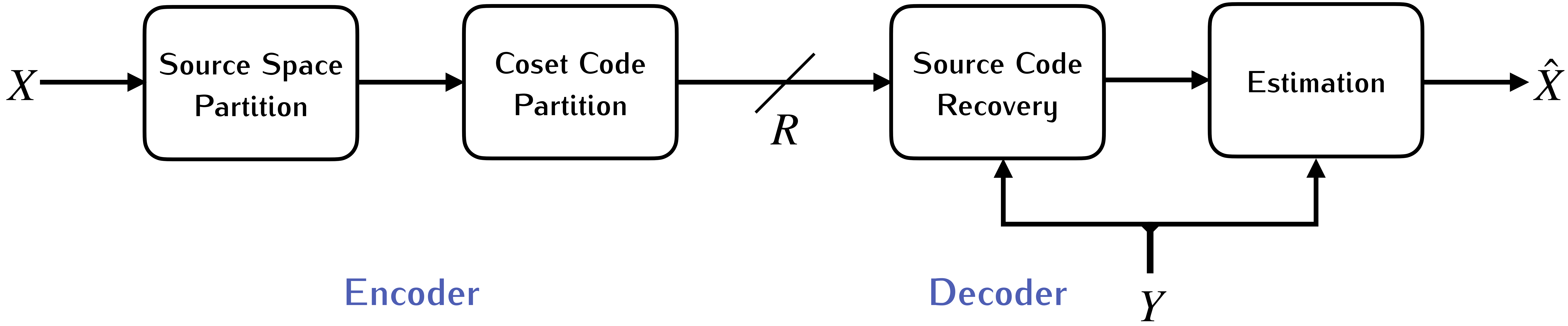

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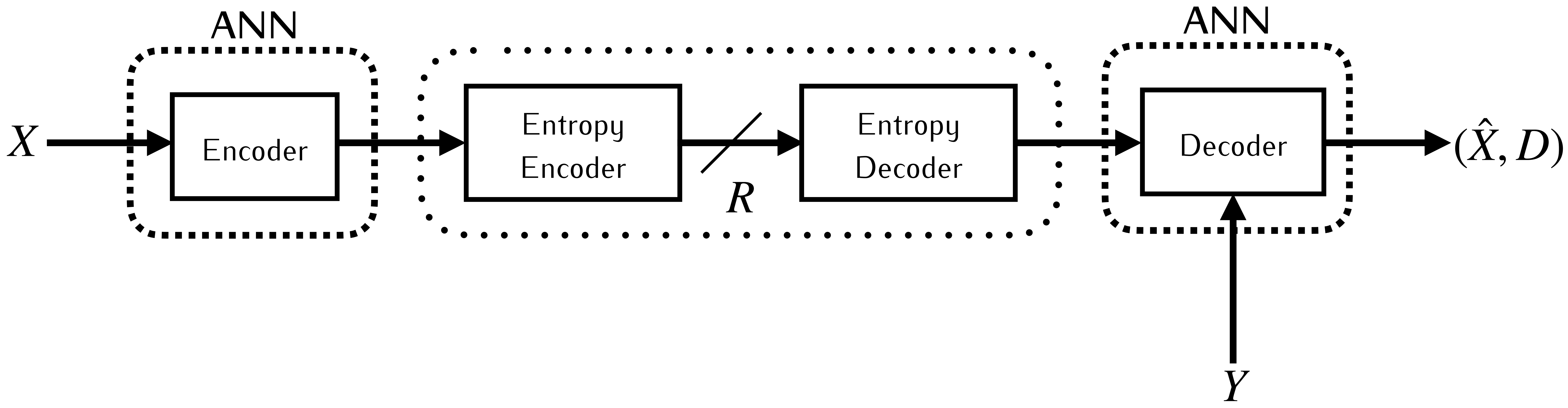

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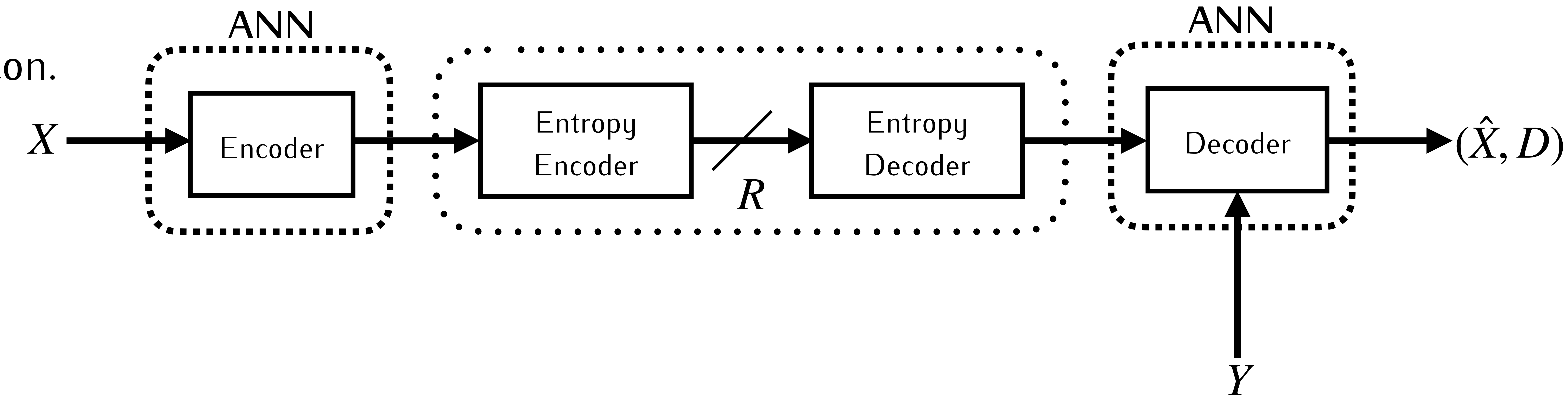

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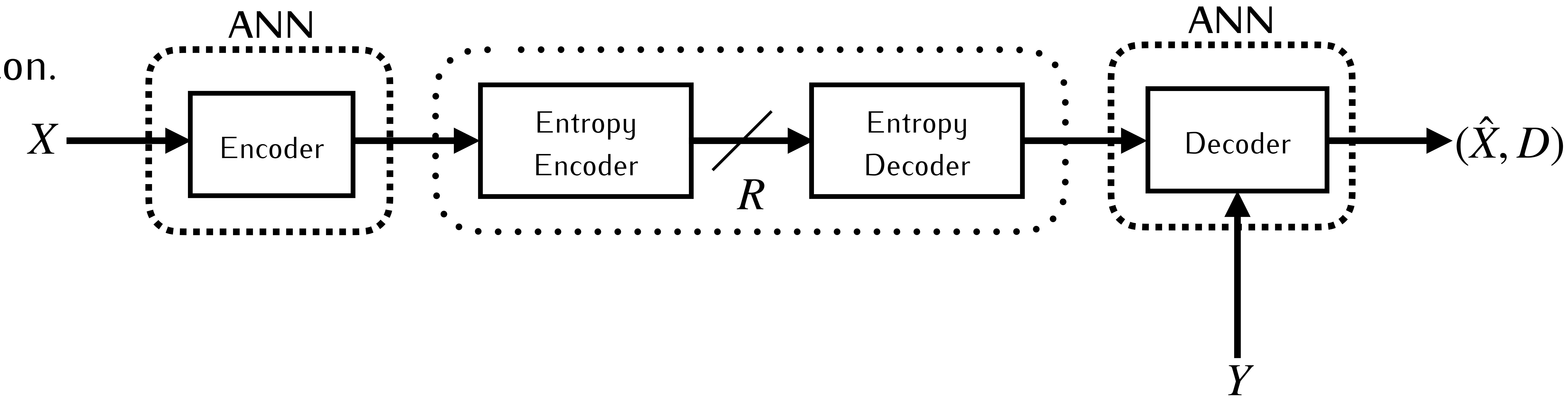

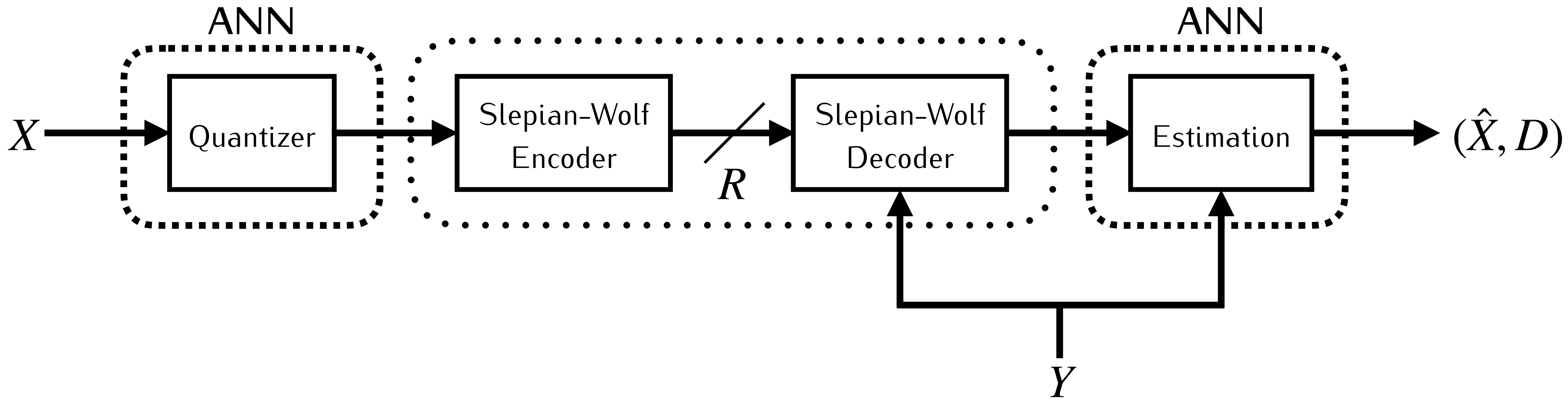

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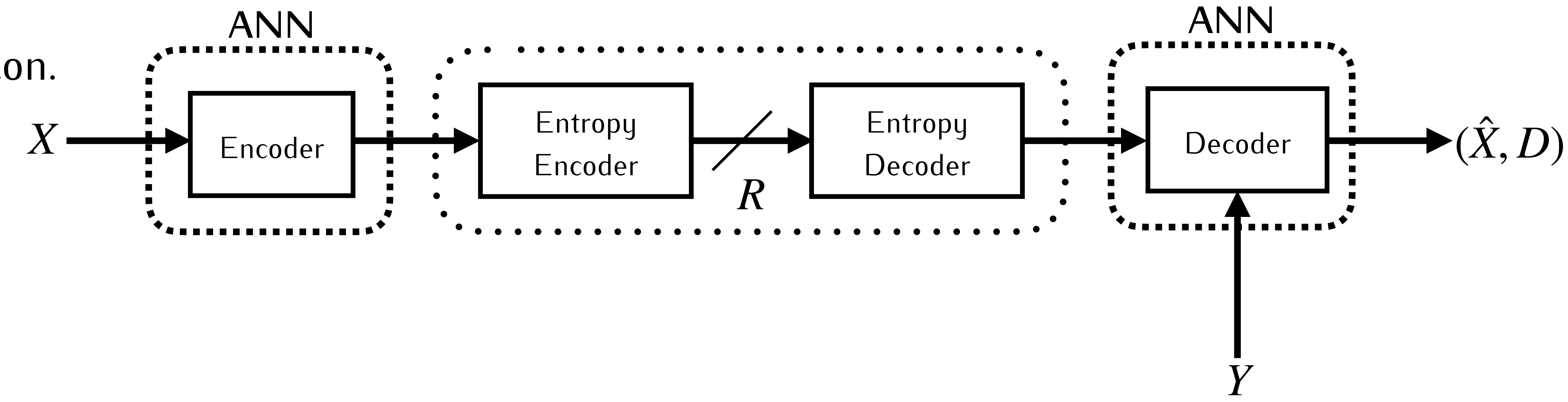

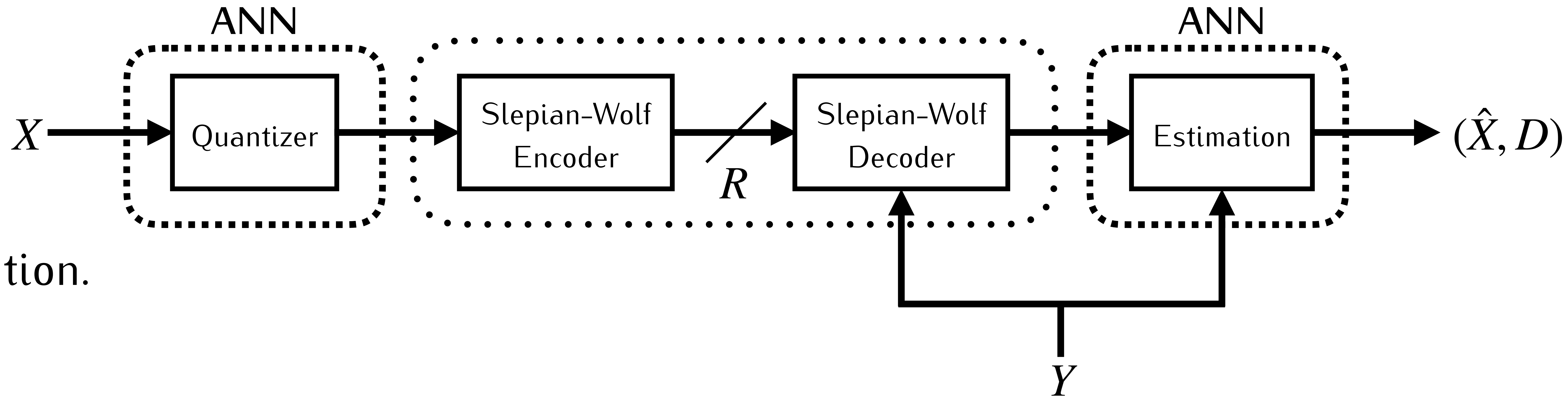

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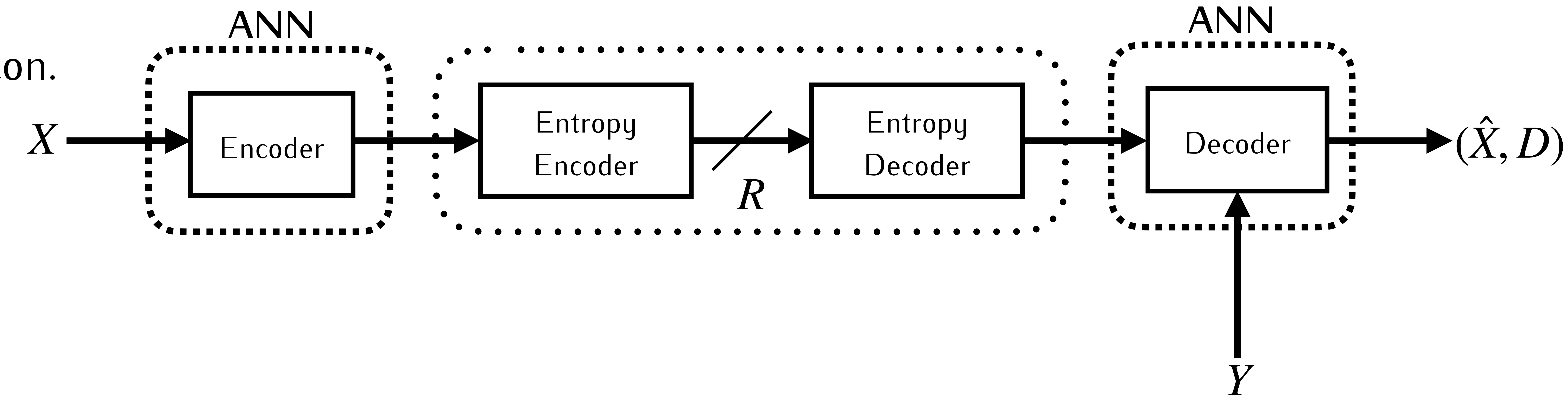

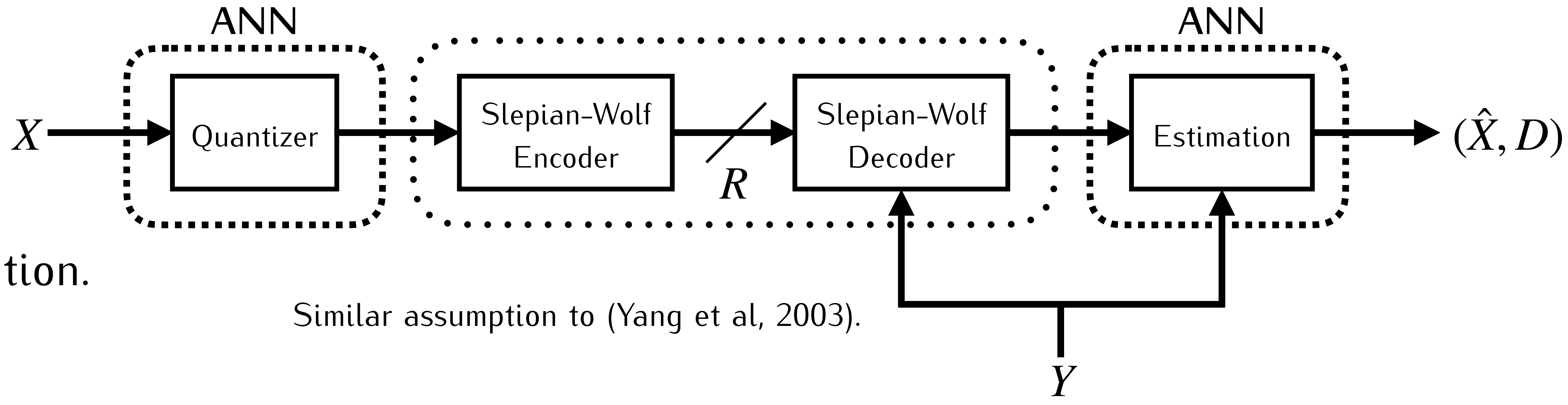

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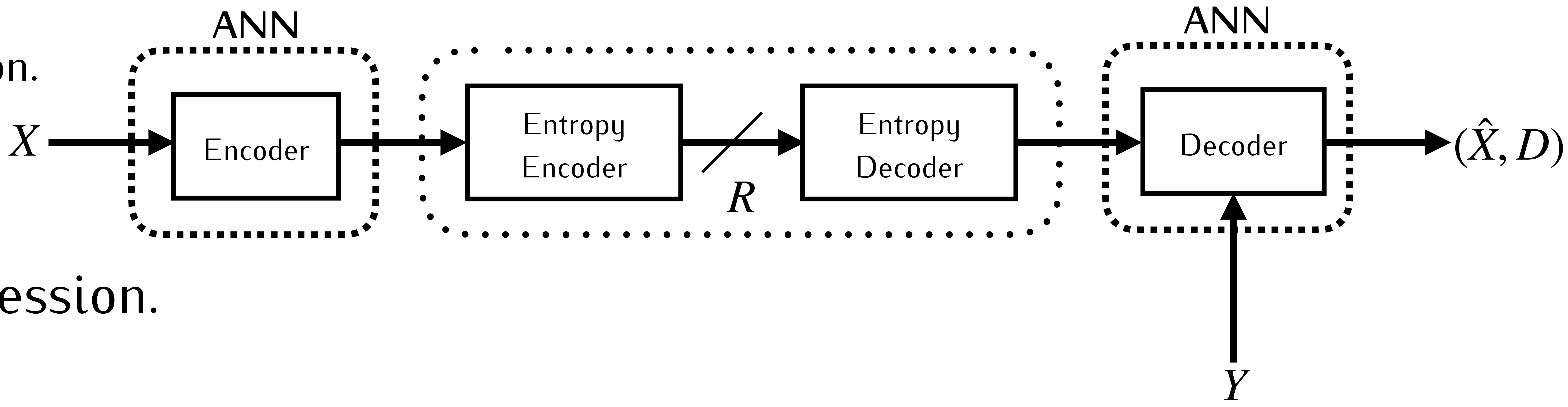

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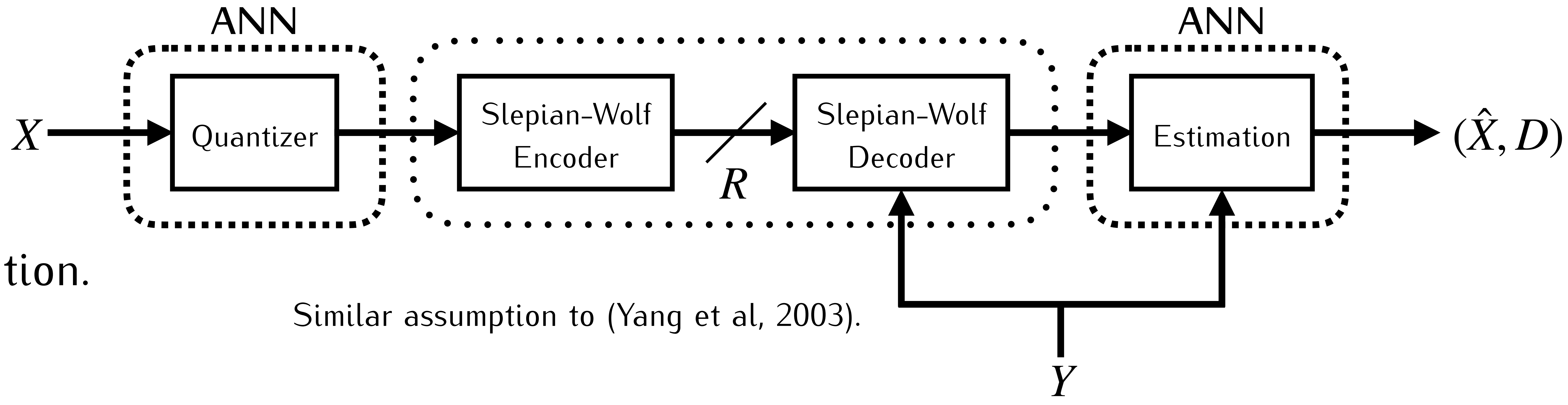

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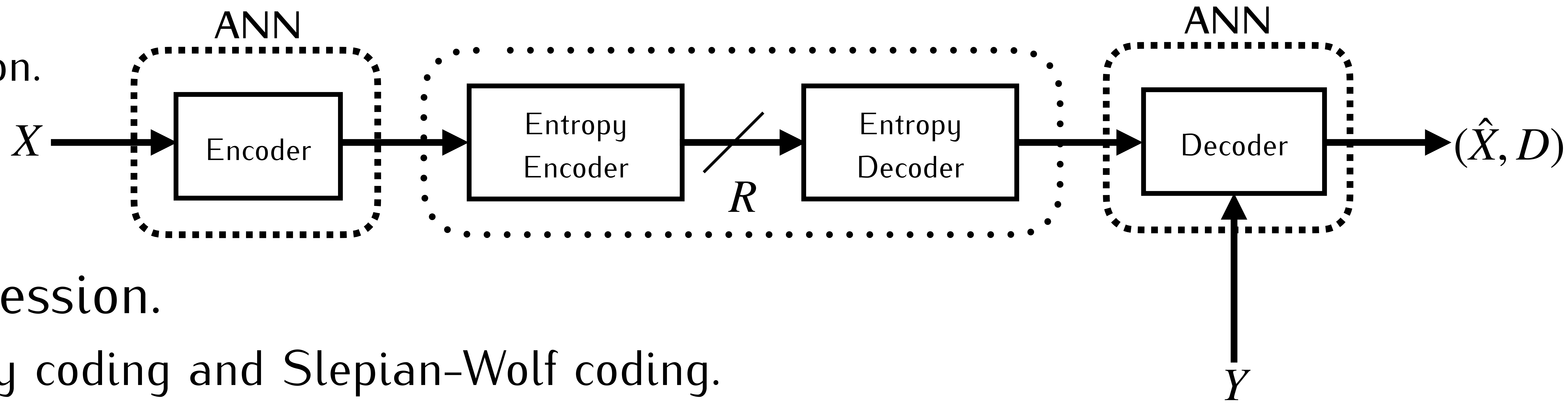

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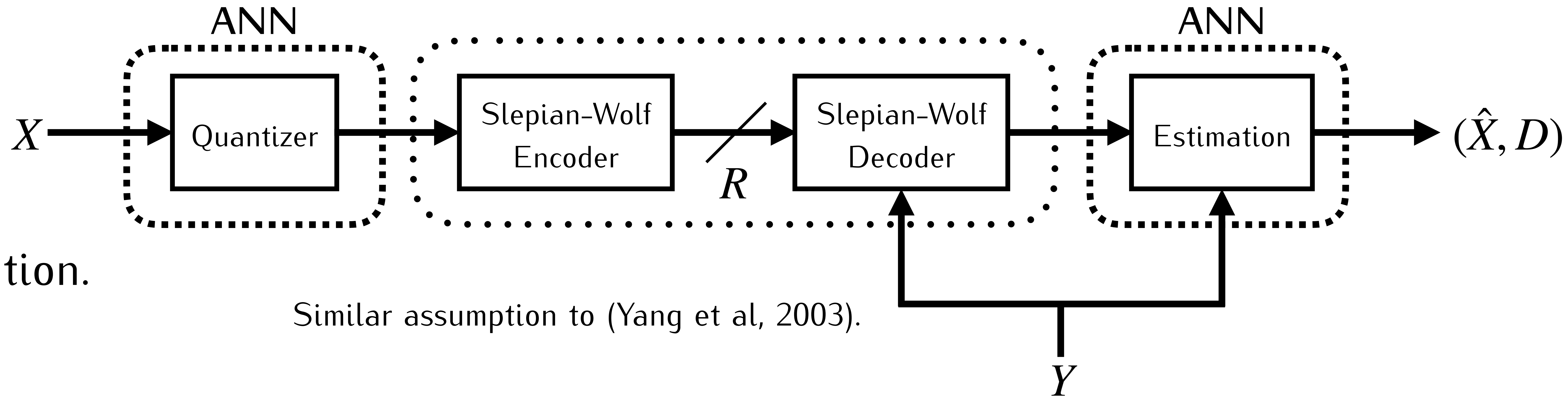

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C. Maddison et al., "The concrete distribution: a continuous relaxation of discrete random variables", *ICLR,* 2017.

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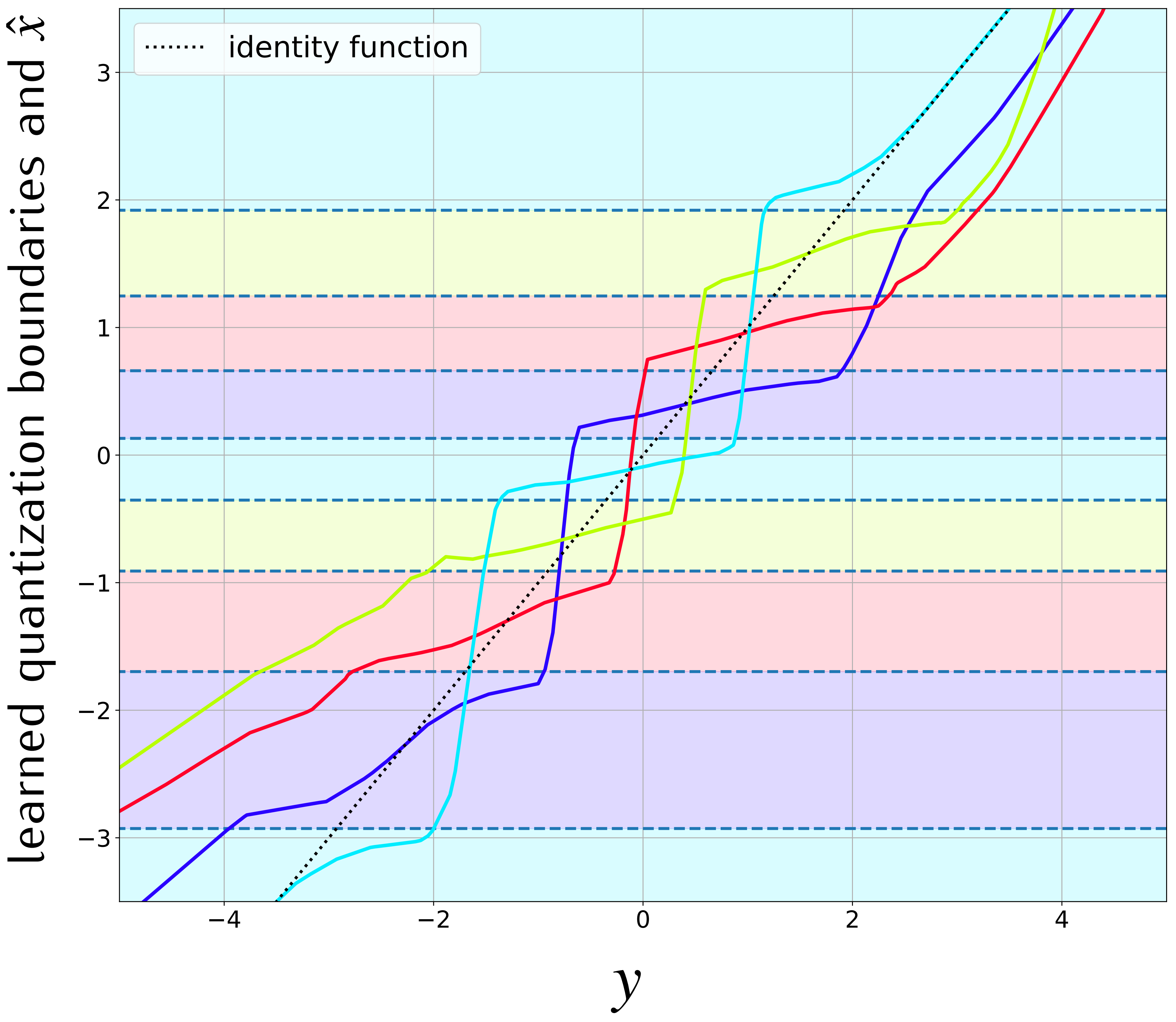

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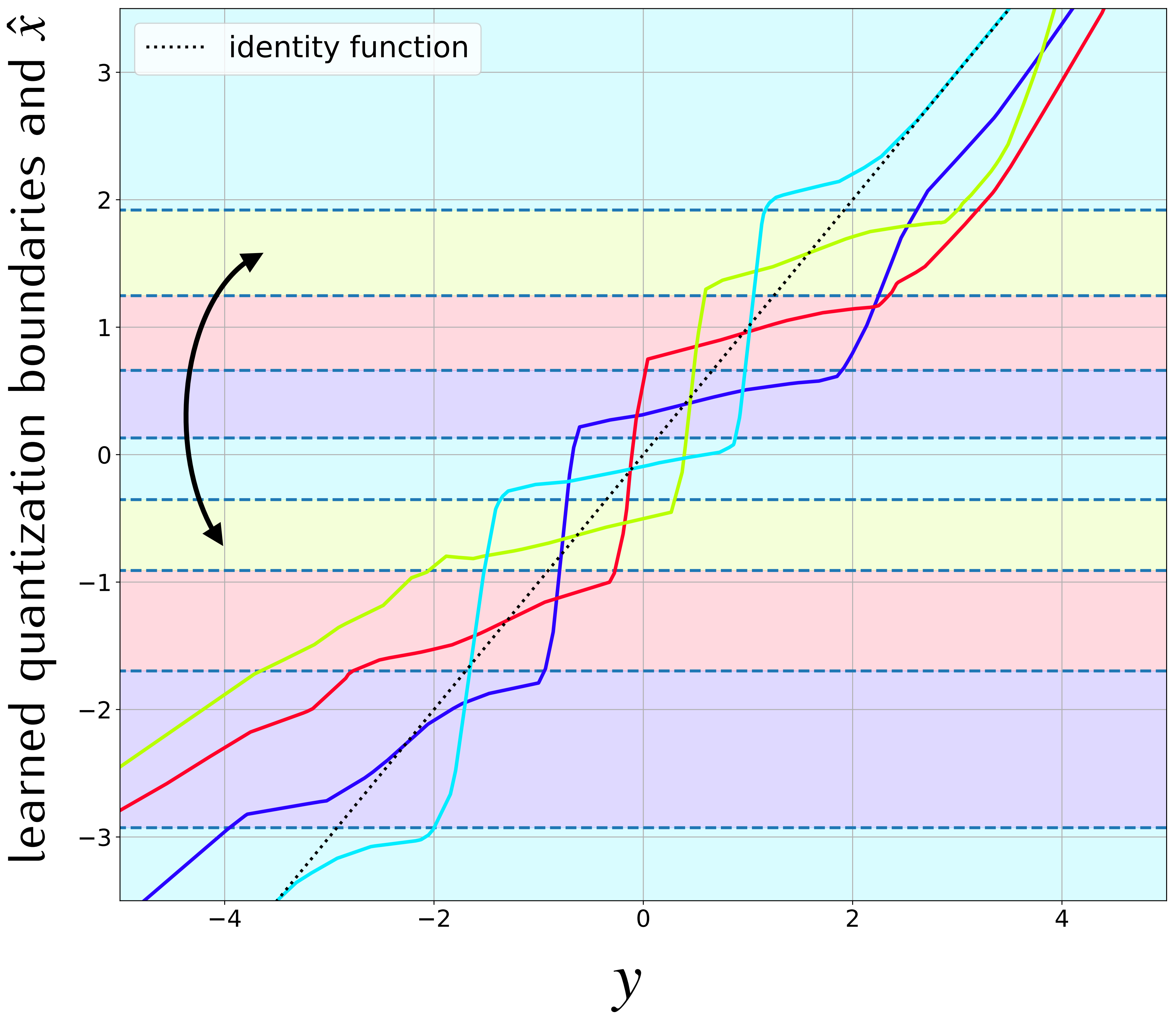

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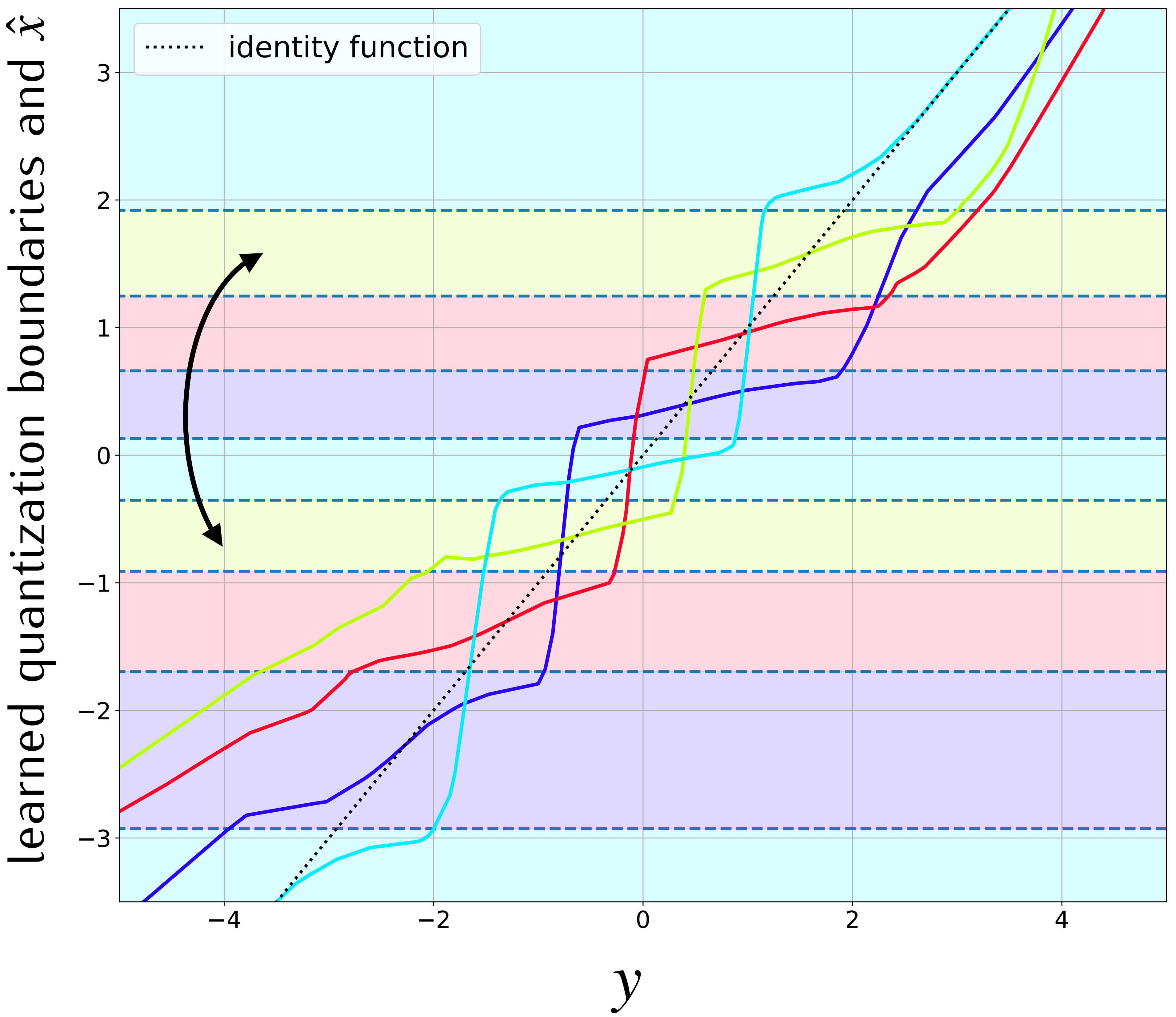

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$X = Y + N$ with $Y \sim N(0,1)$ and $\mathrm{N} \sim N(0,10^{-1})$.

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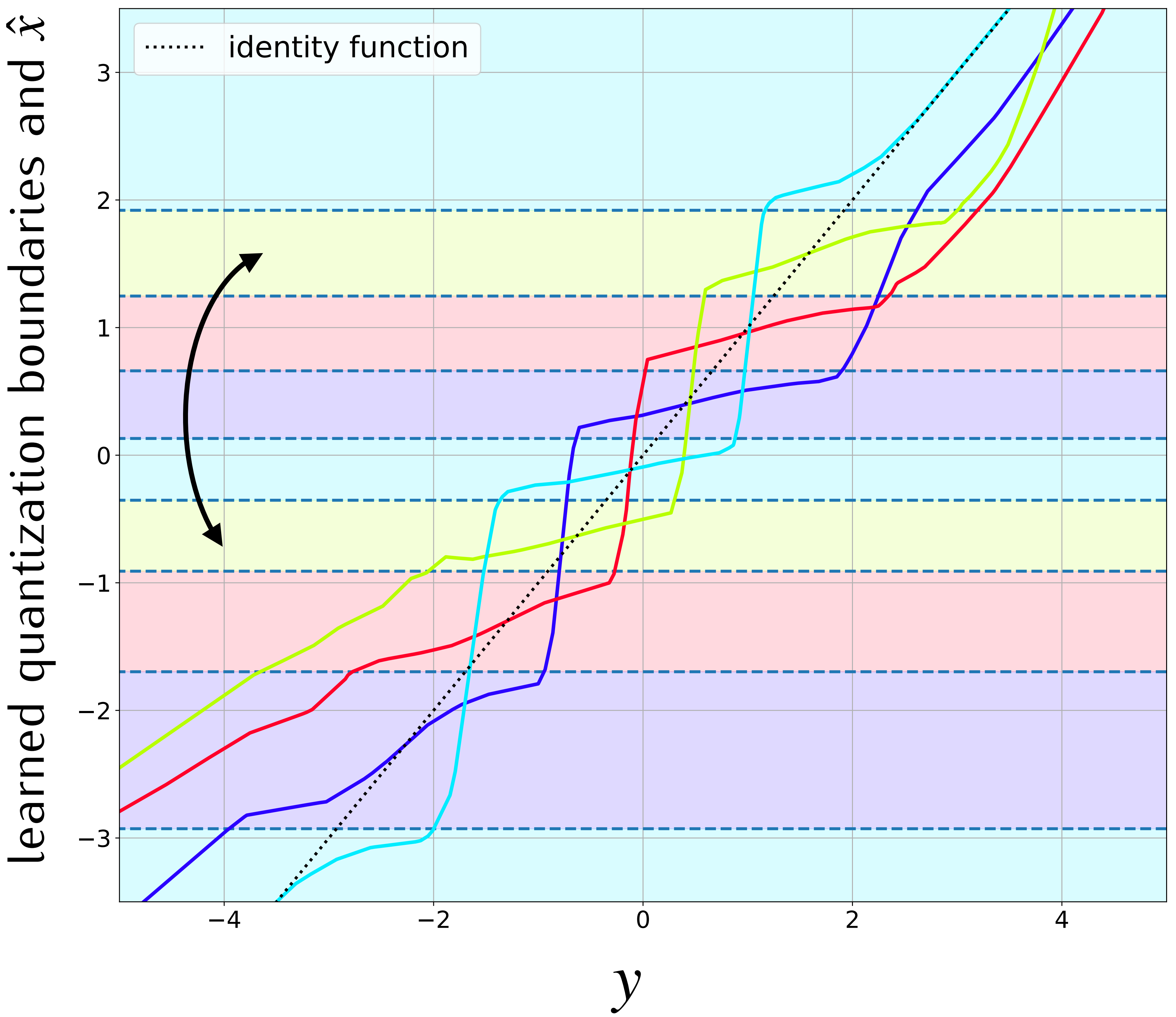

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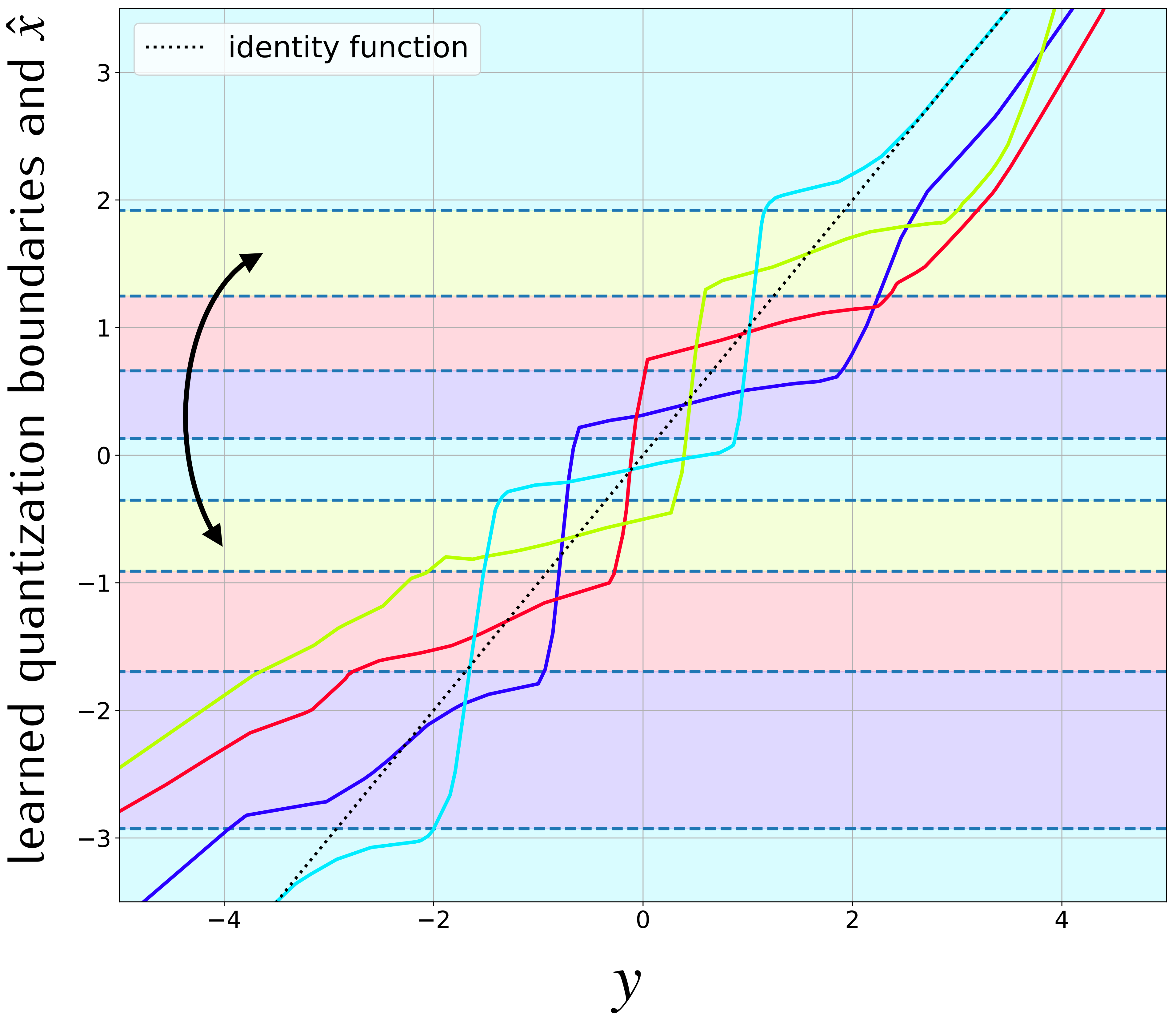

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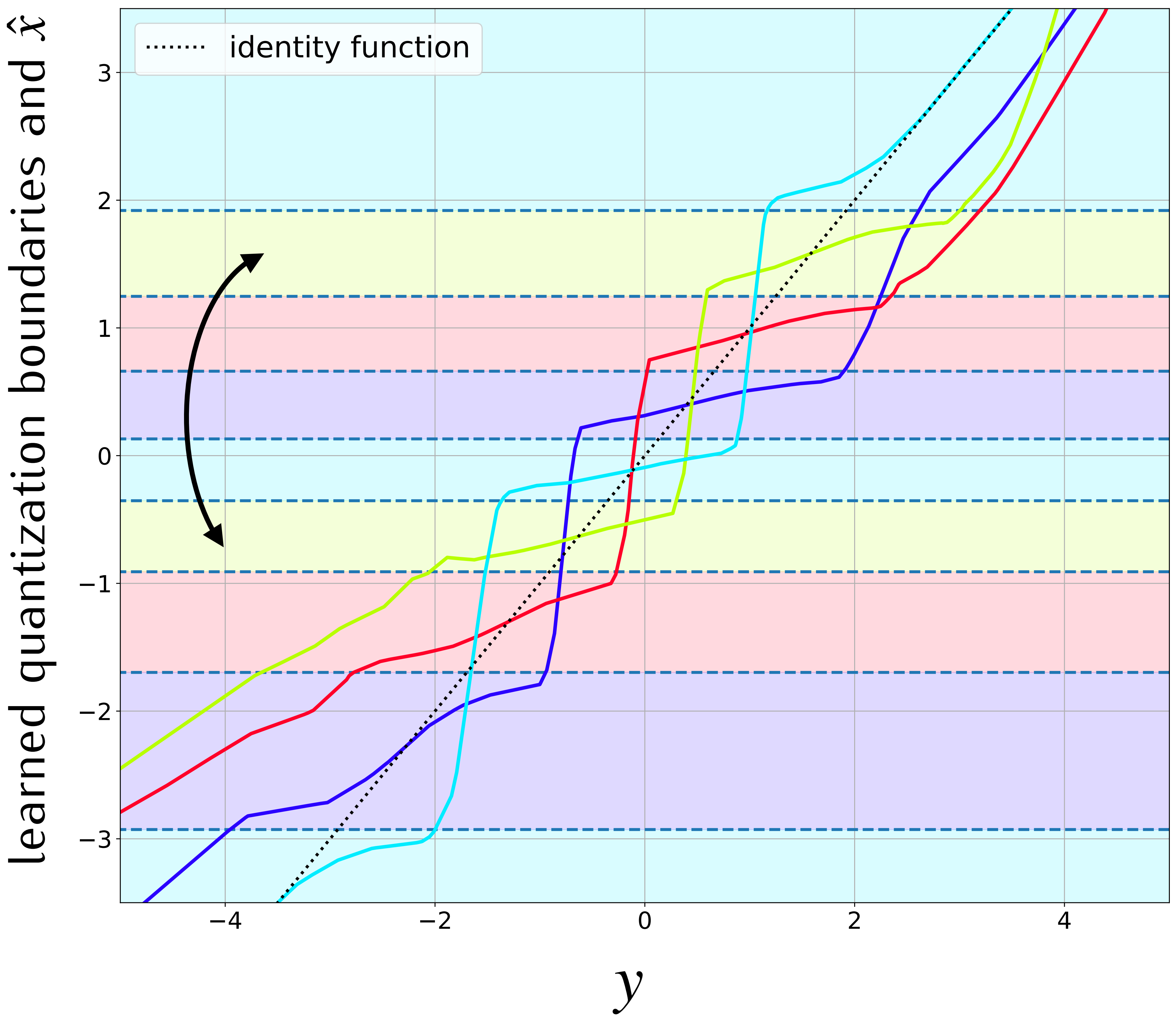

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$$\hat{x} = (1 - \beta) \cdot y + \beta \cdot u,$$

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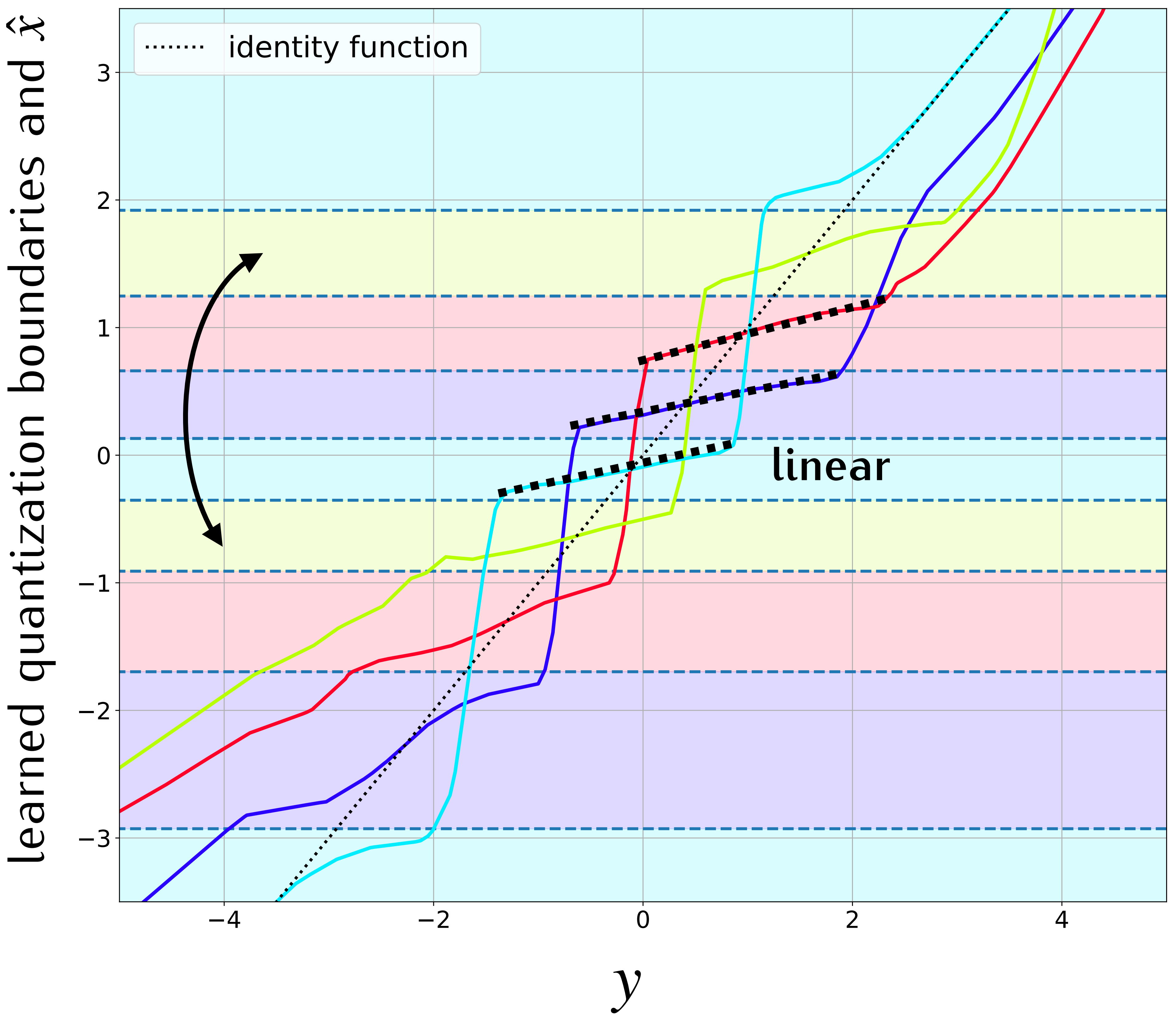

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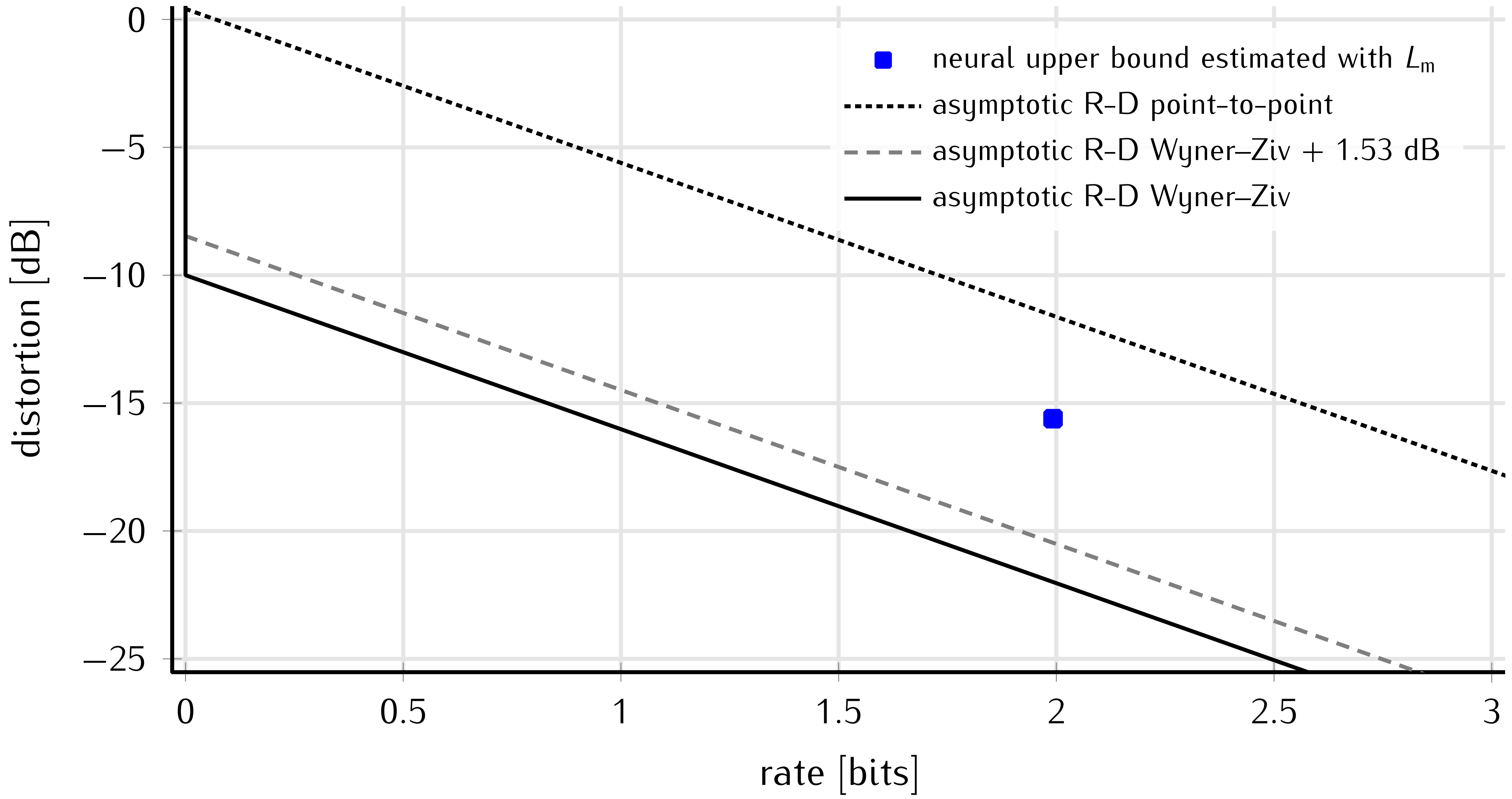

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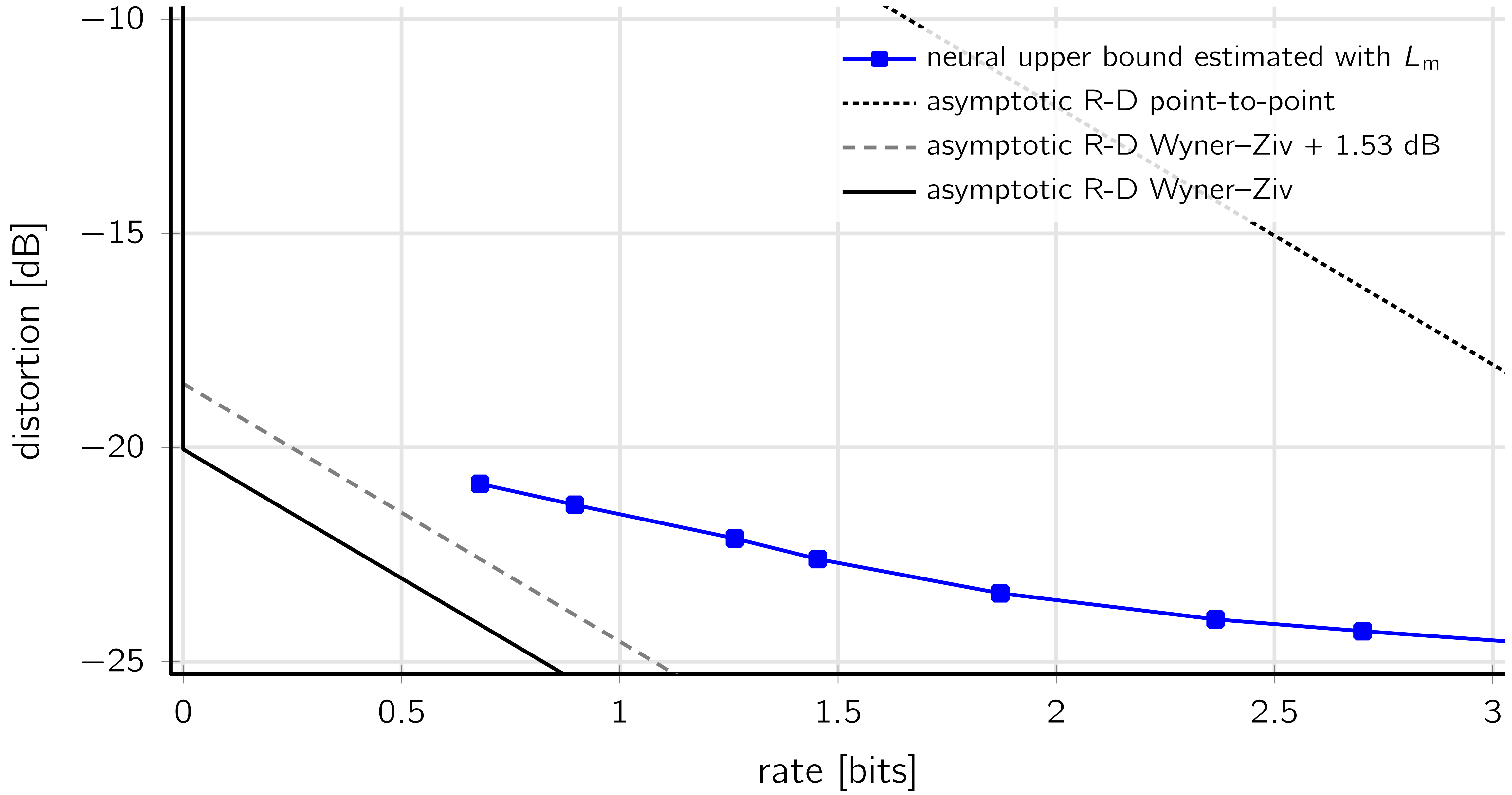

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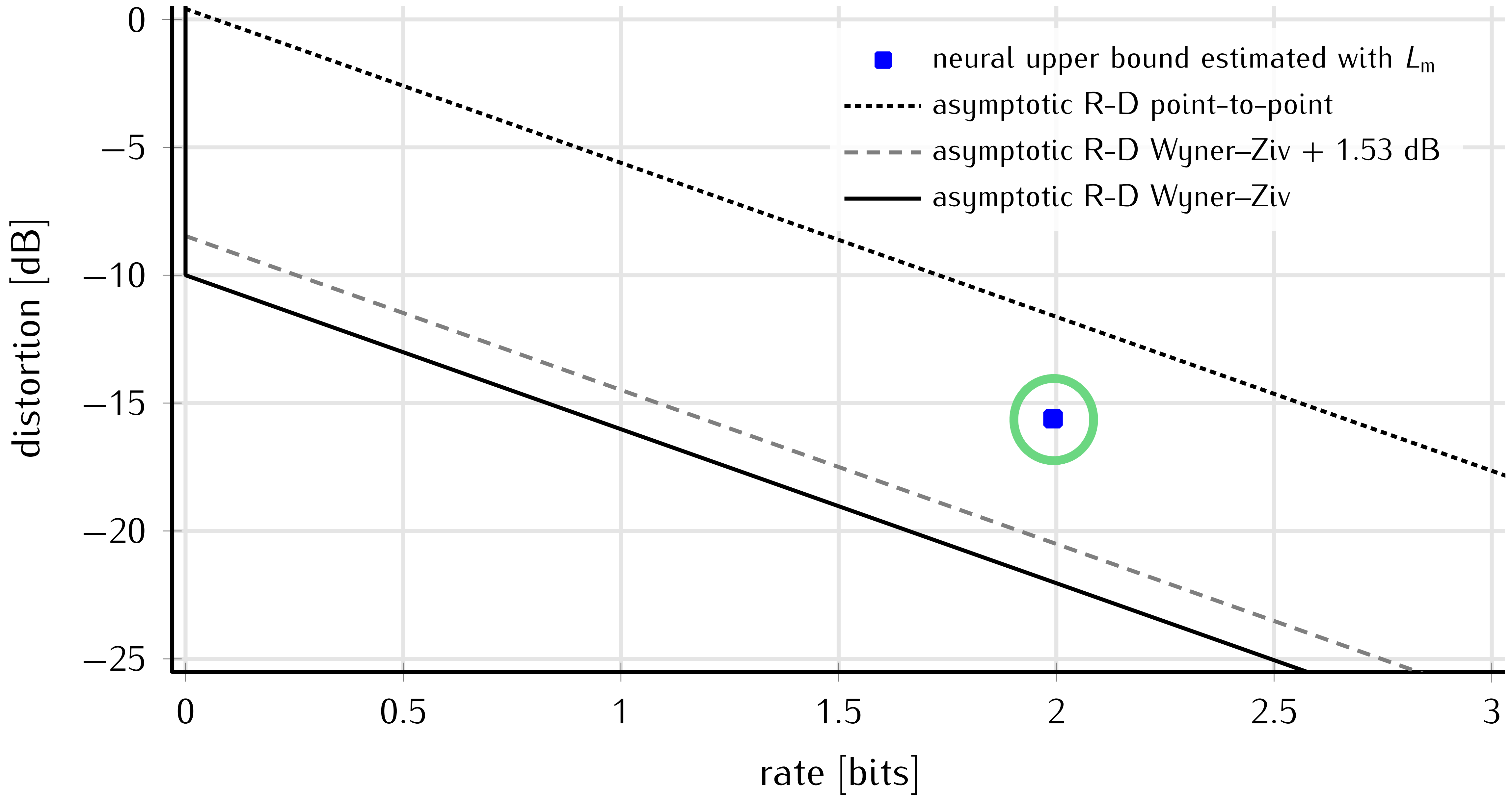

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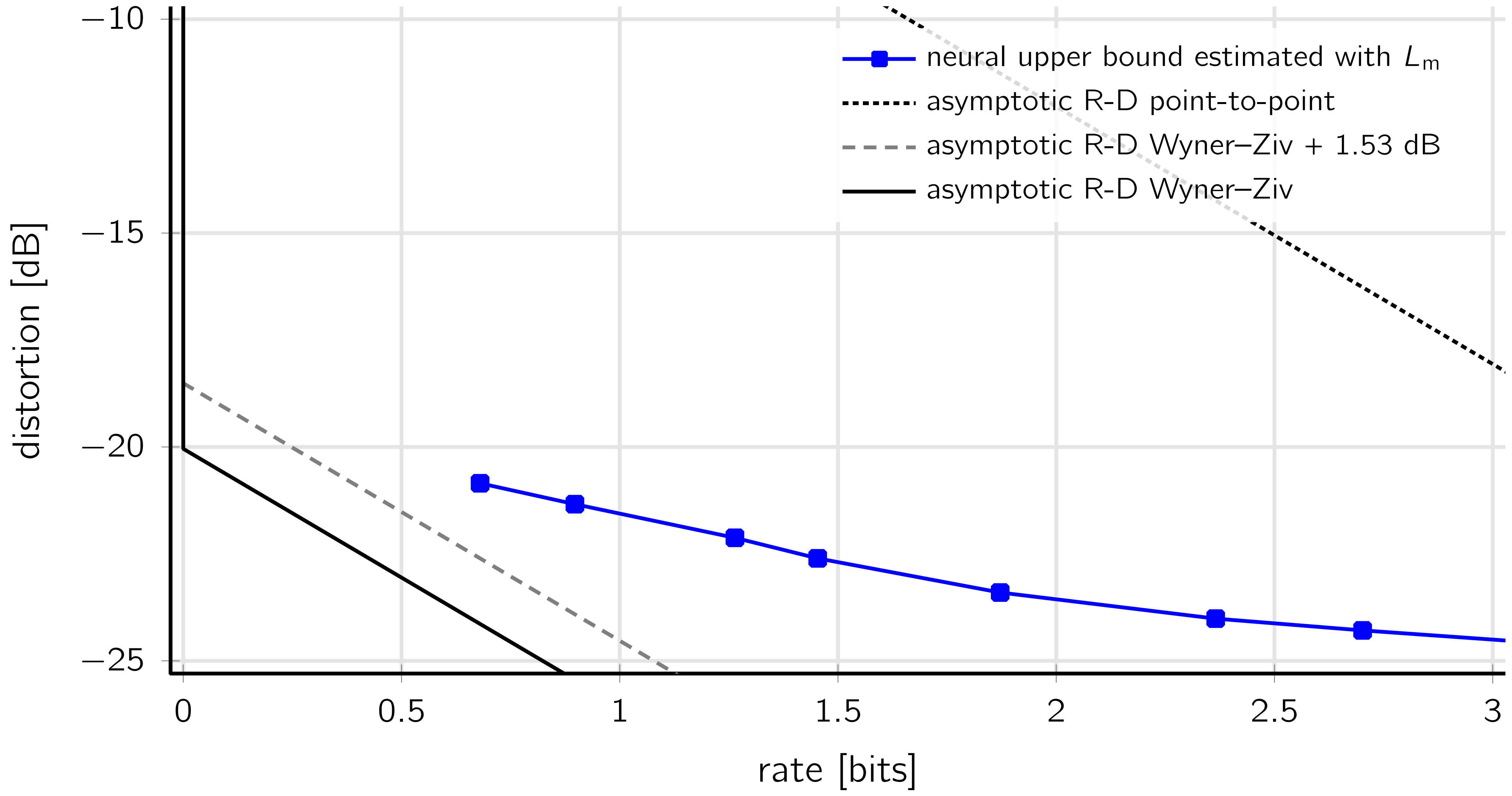

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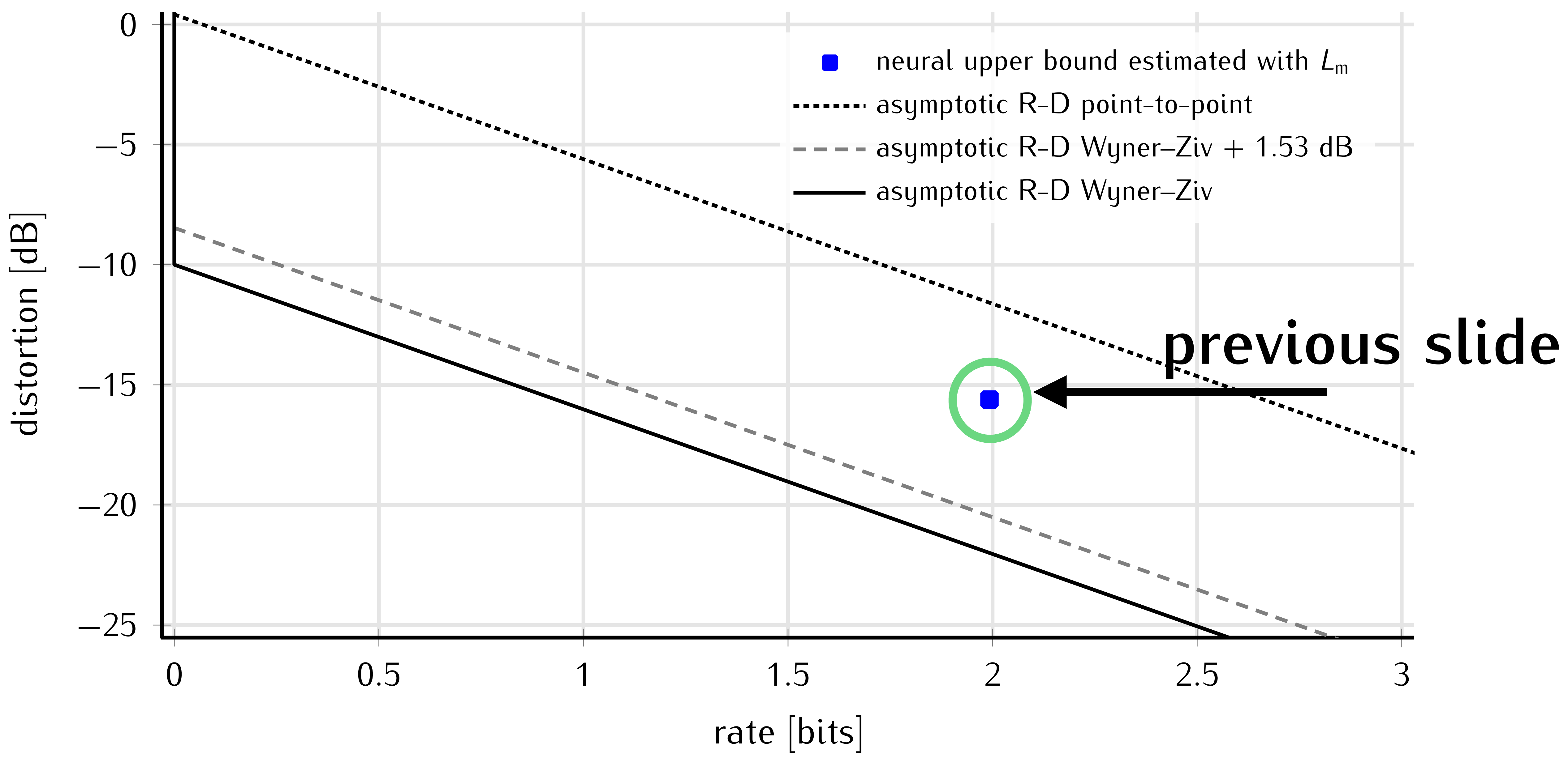

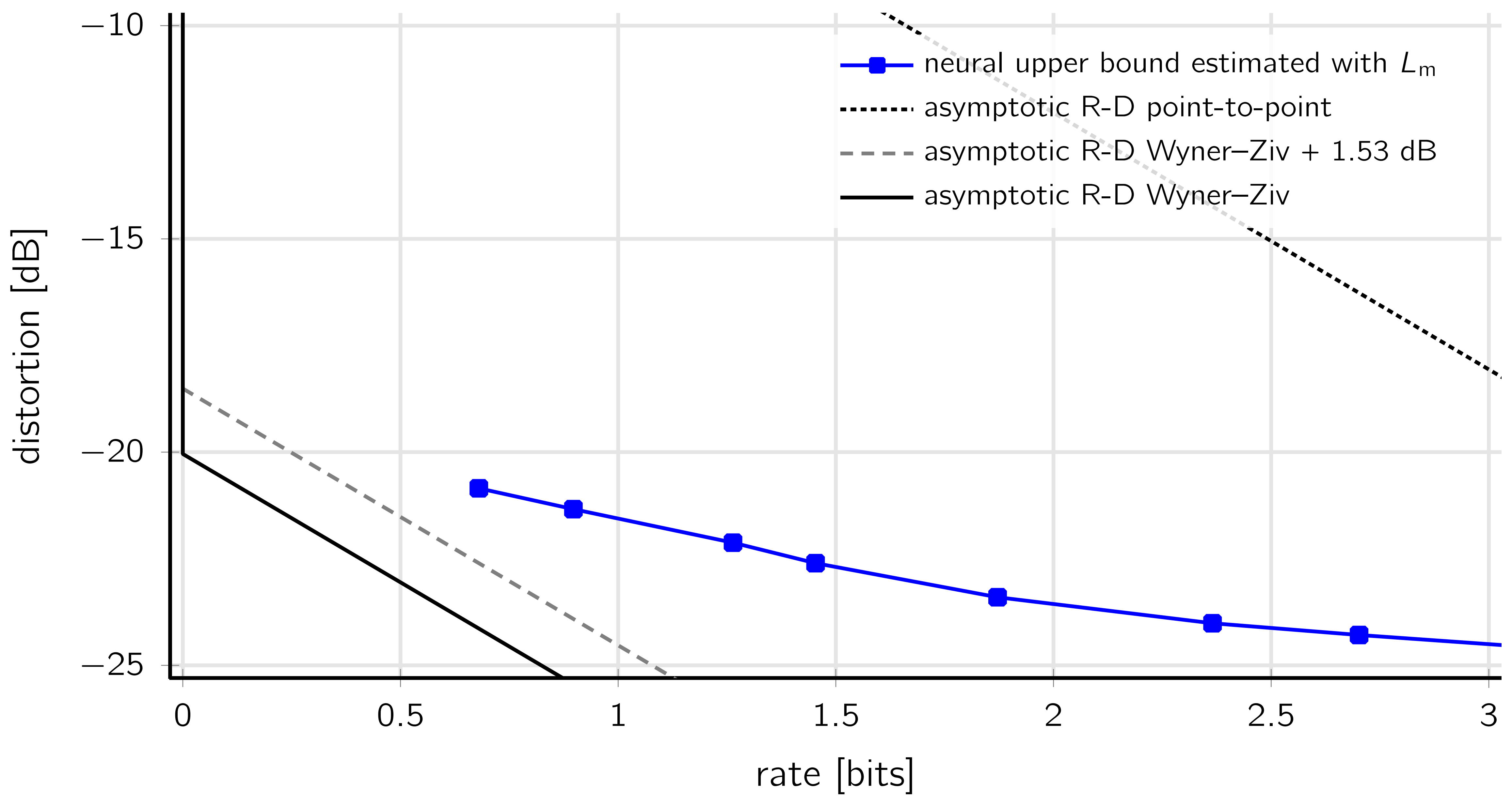

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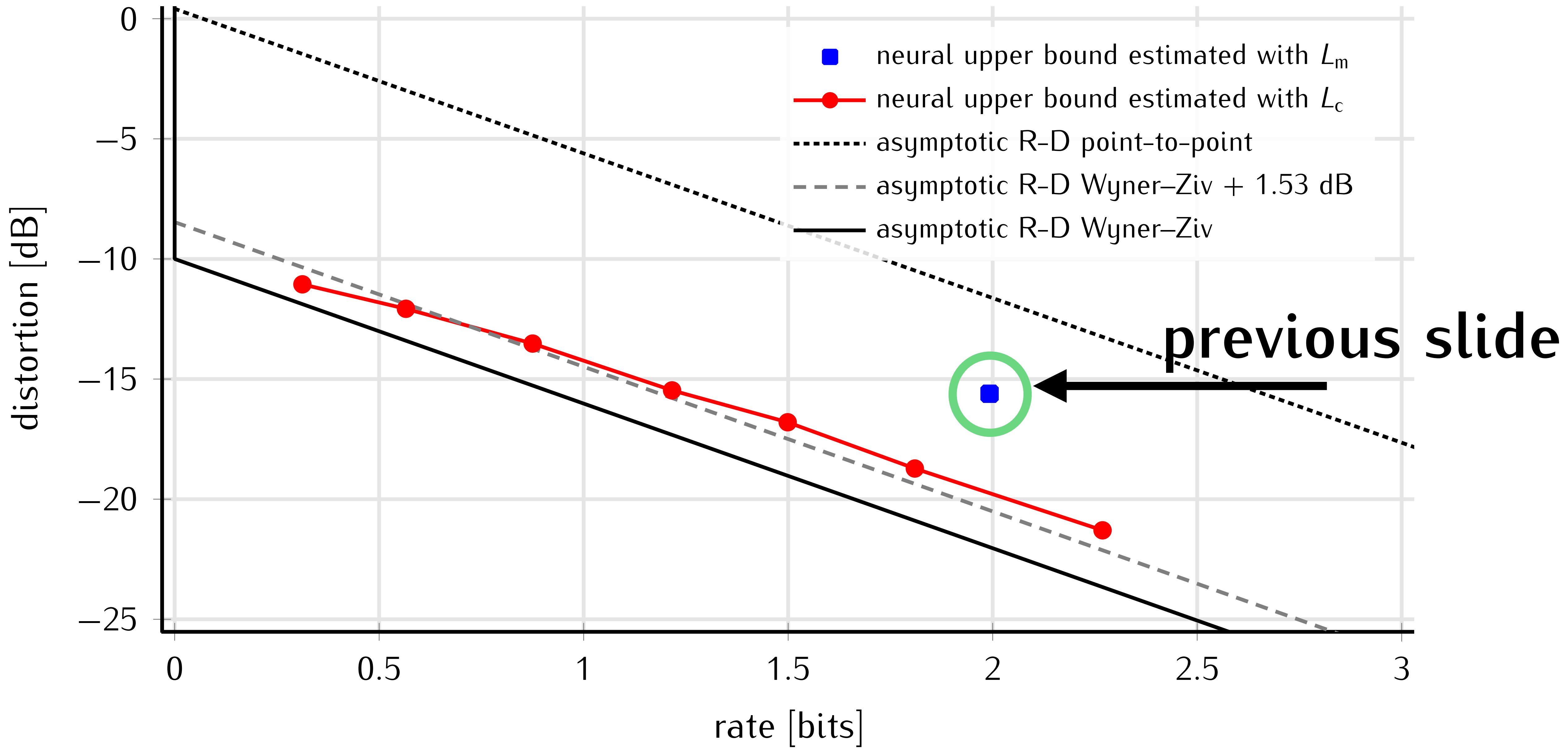

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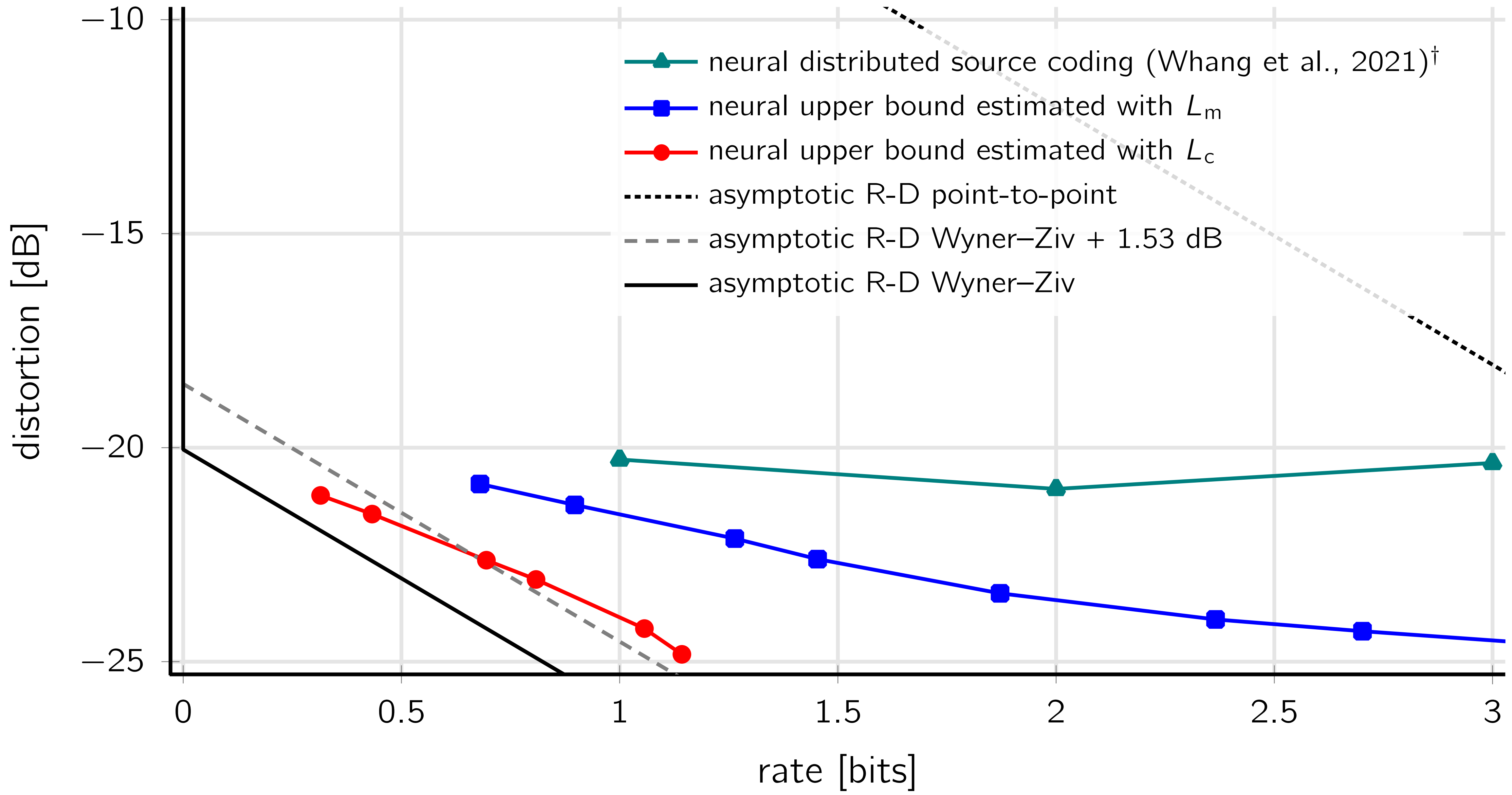

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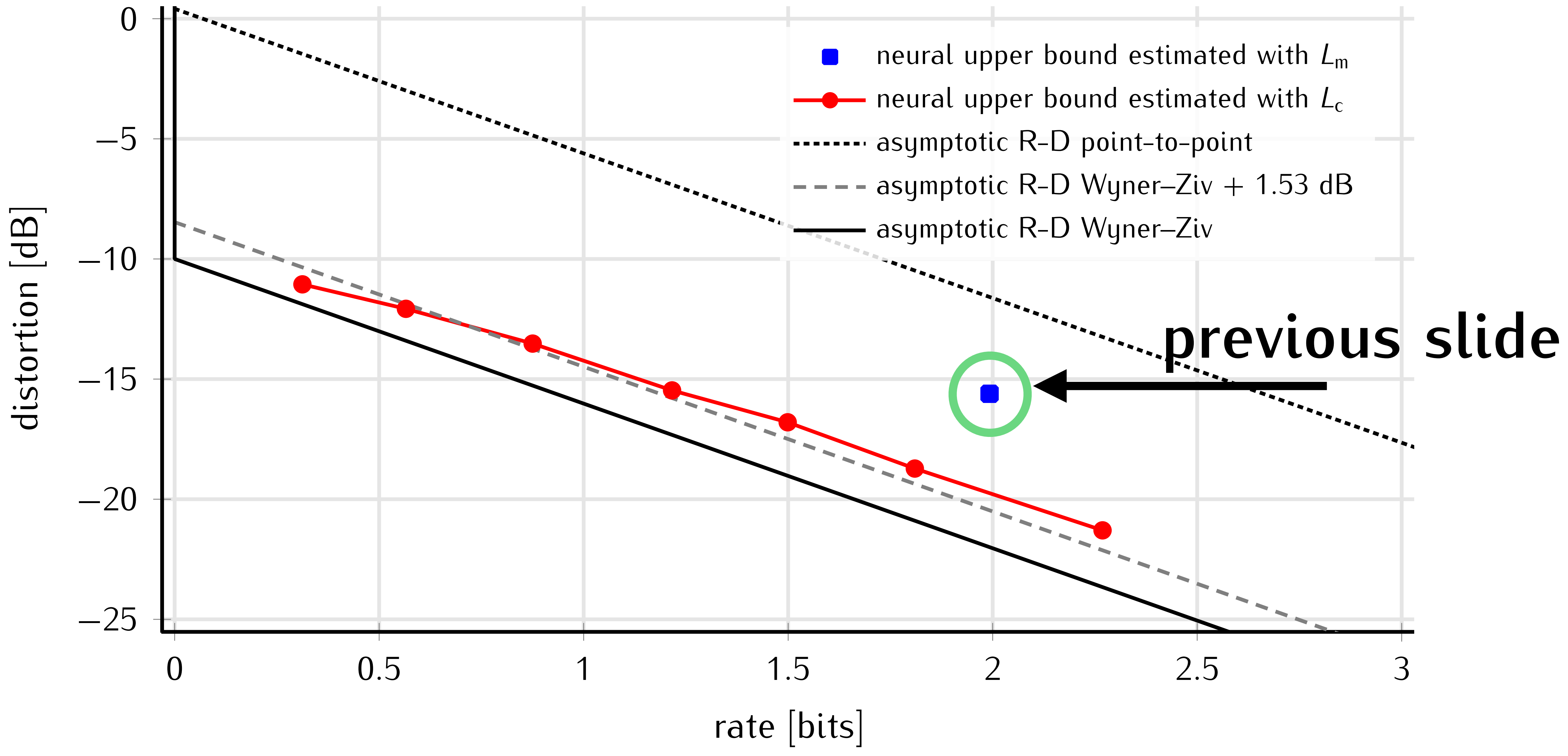

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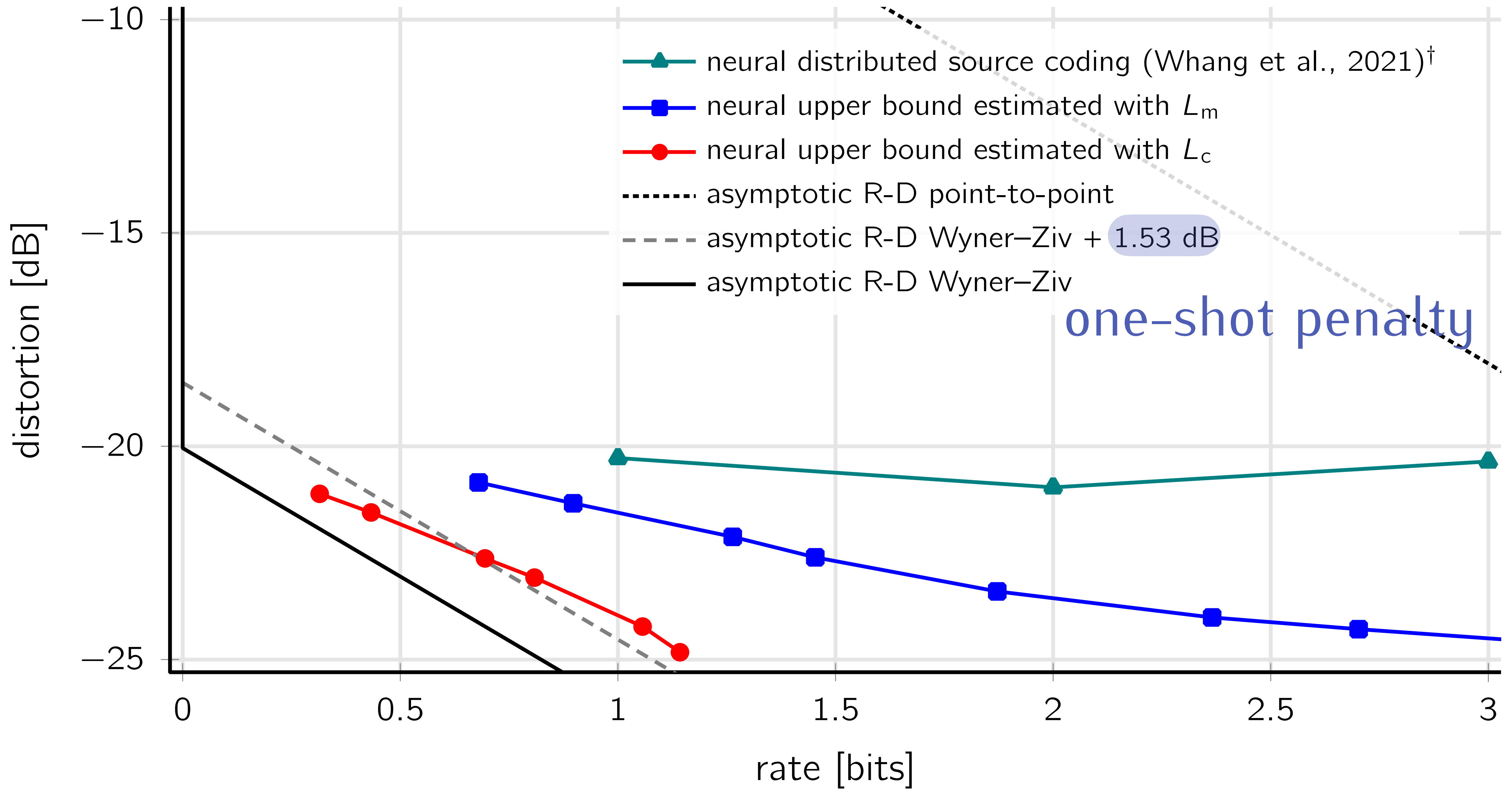

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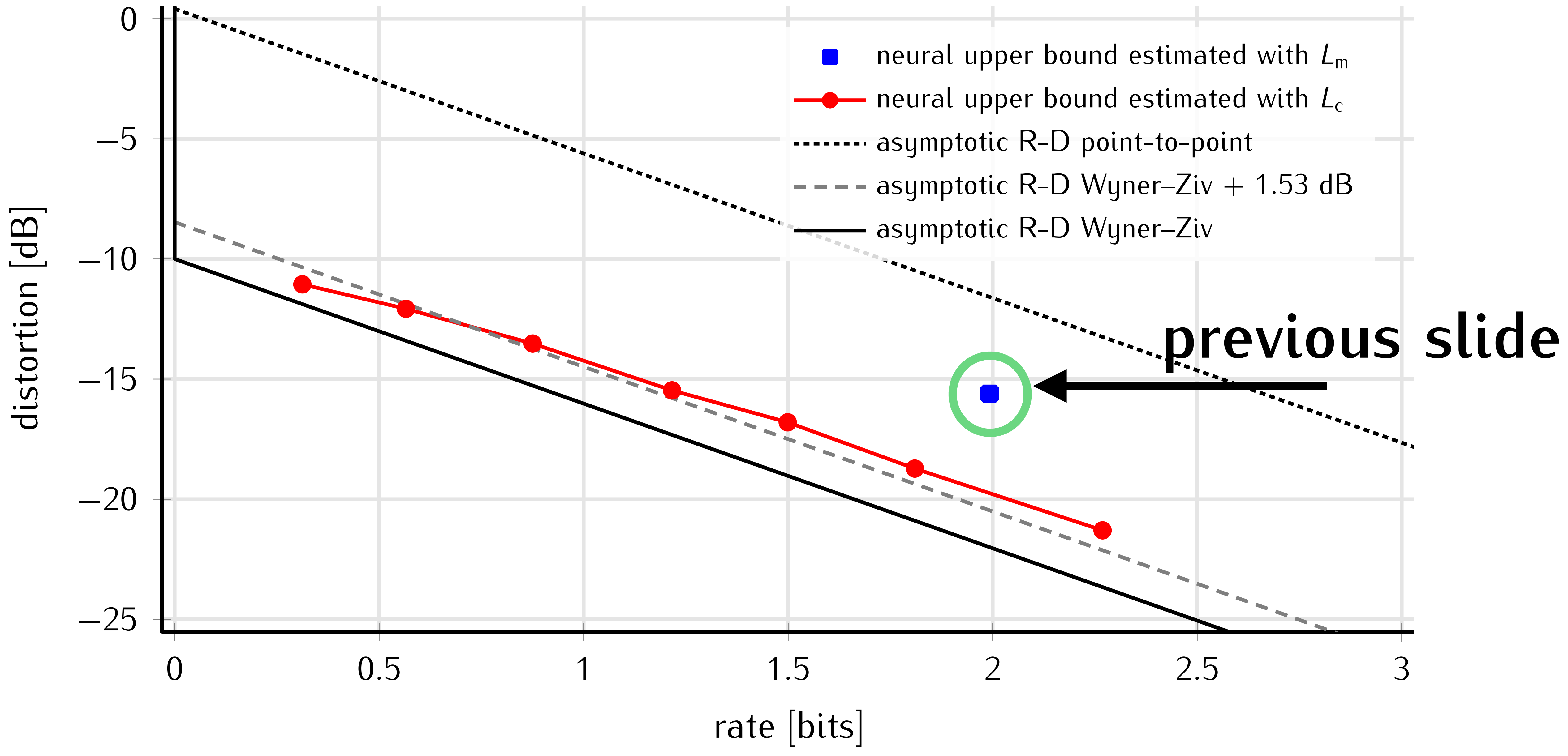

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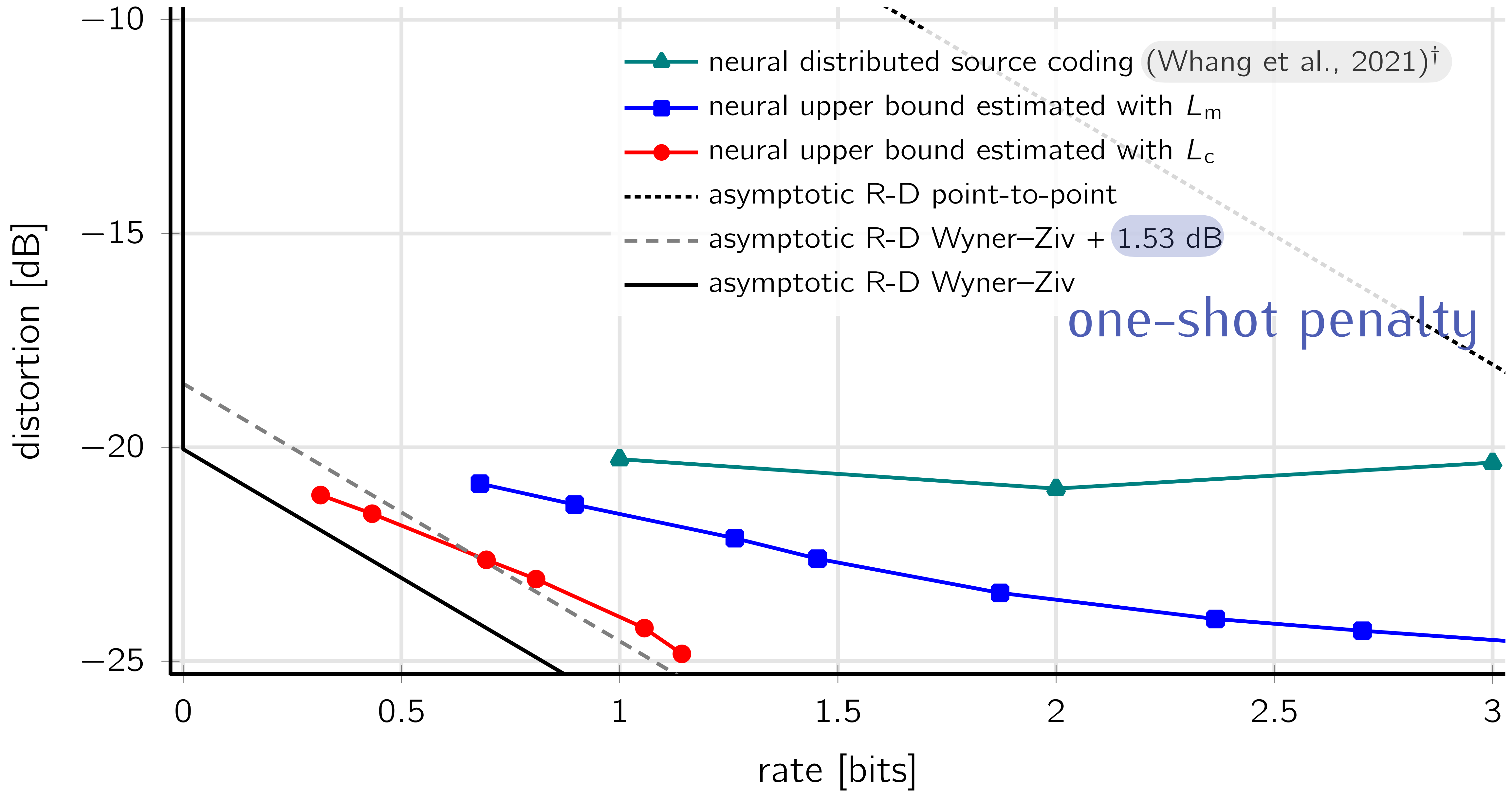

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† J. Whang, A. Acharya, H. Kim, and A. G. Dimakis, "Neural distributed source coding", https://arxiv.org/abs/2106.02797, 2021.

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Thank you. Questions?

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This work was supported in part by NYU Wireless and Google.